ELECTRO MAGNETIC THEORY OF DISPERSION

* Electromagnetic theory of dispersion & anamolous dispersion

Assumption :-

1) There is no appreciable intersection between the atoms of gases molecules.
2) The electric field of the electromagnetic wave induces a dipole moment in the gas molecule
3) Electrons are found to nucleous in an atom by linear restoring force
4) Over the atom or molecules E is constant

$$E = E_0 e^{-i(\omega t - K\cdot r)}$$

$$\approx E_0 e^{-i\omega t} \quad \text{--- (1)}$$

The electric ~~field~~ force acting upon is electron is eE. the electron is also subject to elastic force $(-Kr)$ and damping force $\left(-\alpha \frac{dr}{dt}\right)$. The eqⁿ of motion of electron

$$m\frac{d^2r}{dt^2} = eE - \alpha\frac{dr}{dt} - Kr$$

$$\frac{d^2r}{dt^2} + \frac{\alpha}{m}\frac{dr}{dt} + \frac{K}{m}r = \frac{e}{m}E \quad \text{--- (2)}$$

when $\gamma = \frac{\alpha}{m}$ & $\omega_0^2 = \frac{K}{m}$

The solution of equ^n (2) is given by

$$\sigma = \frac{e/m\, E_0 e^{-i\omega t}}{(\omega_0^2 - \omega^2 - i\gamma\omega)} \quad \text{—— (3)}$$

The polarisation P is equal to the dipole moment ($E\sigma$) per unit volume so that if there are N dispersion electron per unit volume then

$$P = Ne\sigma$$

$$= \frac{Ne^2/m\, E_0 e^{-i\omega t}}{(\omega_0^2 - \omega^2 - i\gamma\omega)} \quad \text{—— (4)}$$

But due to the interaction between the electron the angular velocity of different electron are differected. Therefore the average polarisation of f_K oscillates is given by

$$P = Ne \sum f_K \sigma_K$$

$$= \frac{Ne^2 E}{m} \sum_K \frac{f_K}{(\omega_K^2 - \omega^2 - i\gamma_K\omega)} \quad \text{—— (5)}$$

The electrical ~~polarisation~~ polarisability σ is given by

$$\sigma = \frac{P}{E} = \frac{Ne^2}{m} \sum_K \frac{f_K}{(\omega_K^2 - \omega^2 - i\gamma_K\omega)} \quad \text{—— (6)}$$

~~The electrical polarisation~~

The dielectric constant (K) is given by

$$K = 1 + \frac{1}{\varepsilon_0}\frac{P}{E}$$

$$= 1 + \frac{1}{\varepsilon_0}\frac{Ne^2}{m} \sum_K \frac{f_K}{(\omega_K^2 - \omega^2 - i\gamma_K\omega)} \quad \text{—— (7)}$$

The refractive index is equal to the square root of dielectric constant

$$n^{*2} = 1 + \frac{1}{4\pi\varepsilon_0}\frac{4\pi Ne^2}{m} \sum_K \frac{f_K}{(\omega_K^2 - \omega^2 - i\gamma_K\omega)} \quad \text{—— (8)}$$

This is K/a dispersion formula.

Anomalous dispersion:-

Due to so many natural frequency of the electron the dispersion ~~for~~ is disturbed. For simplicity we assume that there is one natural frequency $\omega_K = \omega_0$

$$n^{*2} = 1 + \frac{1}{4\pi\varepsilon_0} \cdot \frac{4\pi N e^2}{m} \cdot \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

$$n^* = 1 + \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\pi N e^2}{m} \cdot \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega} \quad \text{---} (9)$$

Multiplying numerator and denominator of right hand side by $\omega_0^2 - \omega^2 + i\gamma\omega$ we get

$$n^* = 1 + \frac{1}{4\pi\varepsilon_0} \frac{2\pi N e^2}{m} \left[\frac{\omega_0^2 - \omega^2 + i\gamma\omega}{(\omega_0 - \omega^2)^2 - (i\gamma\omega)^2} \right]$$

$$n^* = 1 + \frac{2\pi N e^2}{4\pi\varepsilon_0 m} \frac{\omega_0^2 - \omega^2}{(\omega_0 - \omega)^2 + (\gamma\omega)^2} + i \frac{2\pi N e^2 \gamma\omega}{4\pi\varepsilon_0 m (\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}$$

Now the complete refractive index

$$n^* = n + iK$$

$$n = 1 + \frac{2\pi N e^2}{4\pi\varepsilon_0 m} \frac{\omega_0^2 - \omega^2}{(\omega_0 - \omega)^2 + \gamma^2\omega^2} \quad \text{---} (1)$$

$$K = \frac{2\pi N e^2}{4\pi\varepsilon_0 m} \frac{\gamma\omega}{(\omega_0 - \omega)^2 + \gamma^2\omega^2}$$