B. Sc.-III Group-A

Derivation of viscosity of gases

Layer by layer flow in liquids is known as laminar flow.
In case of liquids internal friction arises because of inter molecular attractions. Molecules in slow moving layer try to decrease the velocity while the molecules in ~~ffast~~ fast moving layer try to increase the velocity. This results some tangential forces which is required to maintain the uniform flow.

The tangential force F depends on

i, Area of contact between two layers.

ii, velocity gradient $\frac{du}{dz}$ between two layers

$$\therefore \quad F \propto A \cdot \frac{du}{dz}$$

$$\boxed{\therefore \quad F = \eta A \cdot \frac{du}{dz}} \quad \ldots$$

where, η is a constant (viscosity coefficient)
for unit area $A = 1\ m^2/s$

$$\boxed{\therefore \; F = \eta \cdot \frac{du}{dz}} \quad \text{----} \quad (X)$$

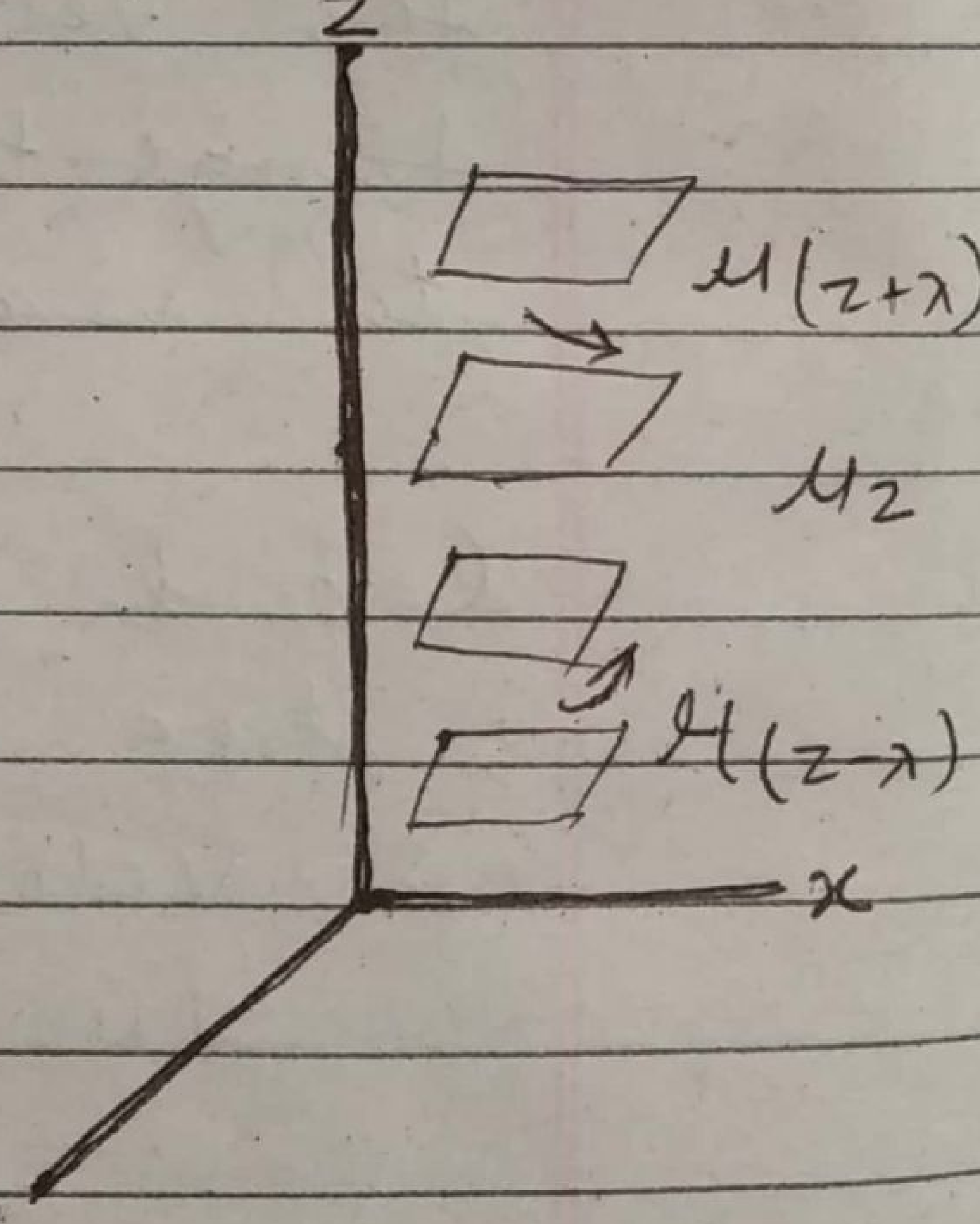
Now suppose,

the gas is made up of plates of series of horizontal layer

The lowest layer is imobilised while the upper layer is having higest velocity U.

∴ Velocity of hight z is given by

$$u_z = \frac{du}{dz} \cdot z$$



at $z = Z$ $\quad u = U$.

so, $$\frac{du}{dz} = \frac{U}{Z} \quad \text{----} \quad i,$$

No. of molecules entering or leaving will be

$$= \frac{1}{6} \rho \bar{c}$$

Where ρ is number density of molecules

$\bar{c}$ is average velocity.

$\therefore$ Momentum of molecules will be given by

$$m u_{(Z+\lambda)} = m \frac{du}{dz}(Z+\lambda) \quad \text{----(ii)}$$

Here, Reduction in momentum

$$m u \downarrow = \frac{1}{6} \rho \bar{c}\, m \frac{du}{dz}(z+\lambda) \quad \text{--(iii)}$$

and increase in momentum

$$m u \uparrow = \frac{1}{6} \rho \bar{c}\, m \frac{du}{dz}(Z-\lambda) \quad \text{----(iv)}$$

$\therefore$ total change in momentum

$$m u \uparrow - m u \downarrow =$$

$$\left[\frac{1}{6} \rho \bar{c}\, m \frac{du}{dz}(z-\lambda)\right] - \left[\frac{1}{6} \rho \bar{c}\, m \frac{du}{dz}(z+\lambda)\right]$$

$$= \frac{1}{6} \rho \bar{c}\, m \frac{du}{dz}\left[z-\lambda-z-\lambda\right]$$

$$= \frac{1}{6} \rho \bar{c}\, m \frac{du}{dz} \cdot 2\lambda$$

Neglecting the sign as it indicates the direction only.

$$= \frac{1}{3} \rho \bar{c} m \frac{du}{dz} \lambda \quad \text{--- (v)}$$

Now Force = momentum/ Area.

unit Area

$\therefore$ Force = momentum

$$\therefore F = \frac{1}{3} \rho \bar{c} m \lambda \frac{du}{dz} \quad \text{--- (vi)}$$

Camparising eqn. (vi) by eqn. (x)

$$\eta \frac{du}{dz} = \frac{1}{3} \rho \bar{c} m \lambda \frac{du}{\cancel{dz}}$$

$$\boxed{\therefore \eta = \frac{1}{3} \rho \bar{c} m \lambda}$$

Viscosity of gases