B.Sc. I
Group-A

Derivation of Kinetic gas equation:

$$PV = \frac{1}{3} mNu^2$$

Solⁿ:

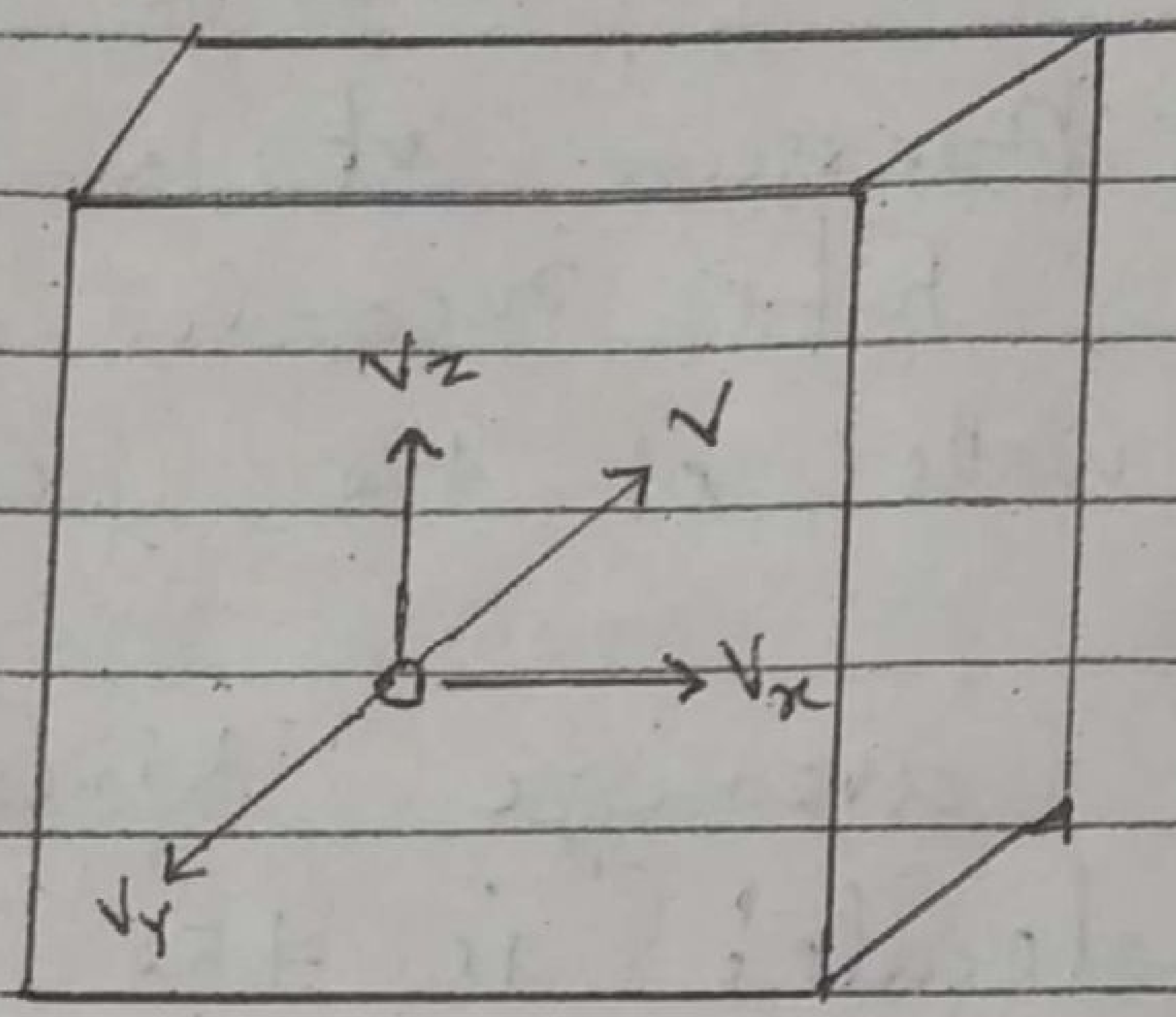


Fig: Resolution of velocity v into components V_x, V_y and V_z.

Let us consider, a certain mass of gas enclosed in a cubic box at a fixed temperature.

the length of each side of the box = l cm
the total number of gas molecule = n
the mass of one molecule = m
the velocity of a molecule = v

steps are :

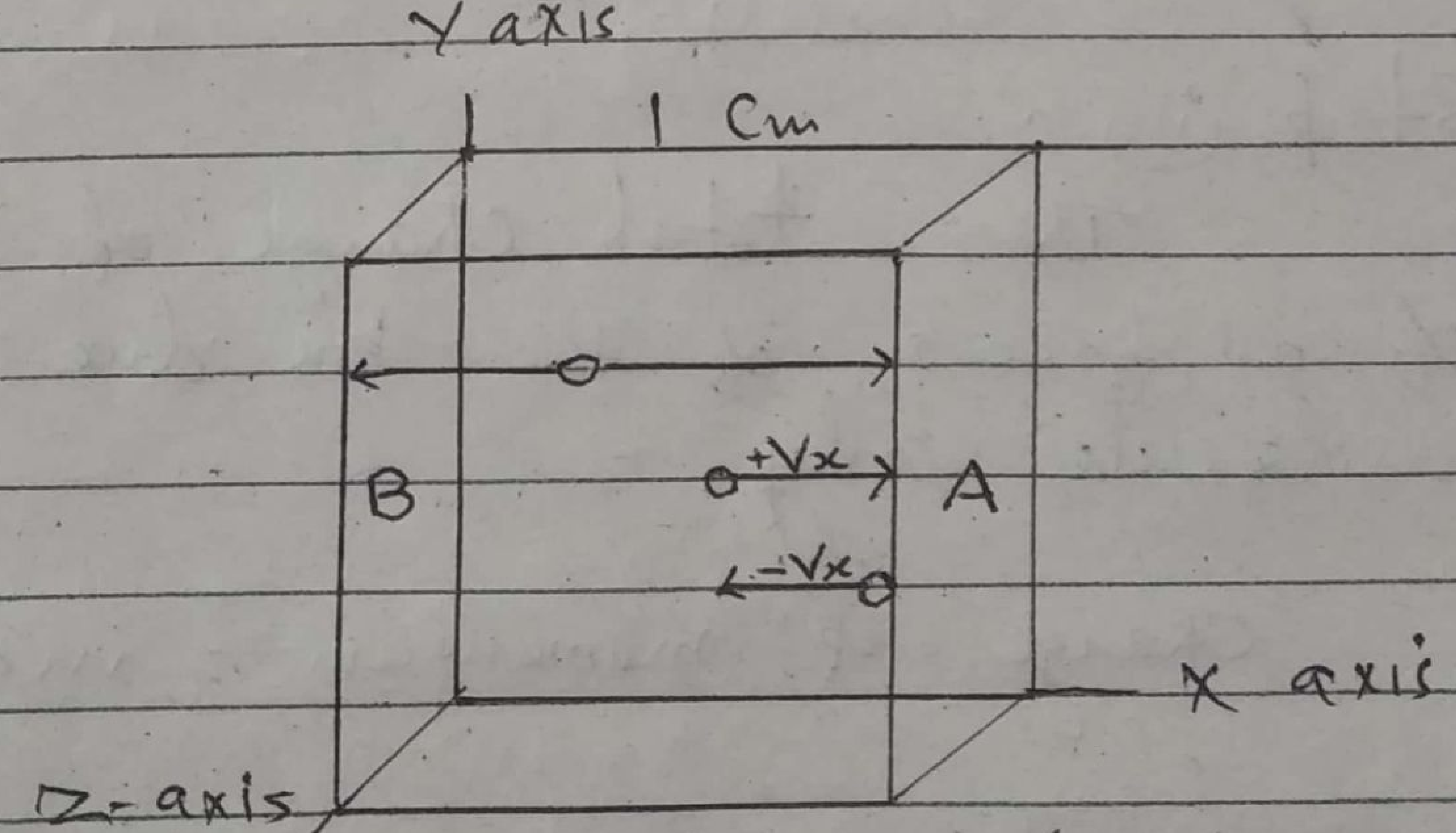
i; : Resolution of velocity v of a Single molecule along x, Y and Z axes.

A molecule of gas can move with velocity v in any direction, and its components are v_x, v_y and v_z along the x, y and Z axies.

Expression

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

Step (ii): The number of collisions Per second on face a due to one molecule



Molecular collisions along x axis

A molecule moving in x direction between opposite faces A and B. The molecule will strike the face A with velocity V_x and rebound with velocity $-V_x$. To hit the same face again, the molecule travel l cm to collide with the opposite face B and then again l cm to return to face A.

$\therefore$ the time between two collisions of face A = $\frac{2l}{V_x}$ seconds

The number of collisions per second on face A = $\frac{V_x}{2l}$

Step (iii):

The total change of momentum of all faces of the box due to one molecule only.

Change of momentum = mass x velocity

the momentum before the impact = mV_x

the momentum after the impact = $m-(-V_x)$

$\therefore$ change of momentum $= mV_x - (-mV_x)$

$= 2mV_x$

On face A number of collision per second of a molecule $= v_x/2l$

$\therefore$ Total change of momentum per second

$$= 2m v_x \times \left(\frac{v_x}{2l}\right)$$

$$= \frac{m v_x^2}{l}$$

Change of momentum on both side (opposite) faces A and B on X-axis will be double

i.e. $2mv_x^2/l$

Similarly for Y-axis $2mv_y^2/l$

for Z-axis $2mv_z^2/l$

On all faces change of momentum will be

$$= \frac{2mv_x^2}{l} + \frac{2mv_y^2}{l} + \frac{2mv_z^2}{l}$$

$$= \frac{2m}{l}\left(v_x^2 + v_y^2 + v_z^2\right)$$

$$= 2mv^2/l \qquad (\because v^2 = v_x^2 + v_y^2 + v_z^2)$$

Step (iv) Total change of momentum due to impacts of all the molecules on all faces of the box.

Suppose N molecules is moving with different velocity v_1, v_2, v_3 --- etc.

$$= \frac{2m}{l}\left(v_1^2 + v_2^2 + v_3^2 + \cdots\right)$$

Multiplying and dividing by N

$$= \frac{2mN}{l}\left[\frac{v_1^2 + v_2^2 + v_3^2 + \cdots}{n}\right]$$

$$= 2mNu^2/l \qquad \because u^2 = \text{mean square velocity}$$

Step (v) : Calculation of pressure from change of momentum :

Total force = Total area x Pressure

$$\text{Force} = \frac{2mNu^2}{l}$$

$$P = \frac{2mNu^2}{l} \times \frac{1}{6l^2}$$

$$P = \frac{1}{3}\frac{mNu^2}{l^3}$$

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$\because l^3 = \text{volume } (V)$

$$\therefore \boxed{PV = \frac{1}{3} m N u^2}$$

This is known as Kinetic gas equation.

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