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TRANSMISSION OF PARTICLE THROUGH A RECTANGULAR POTENTIAL BARRIER

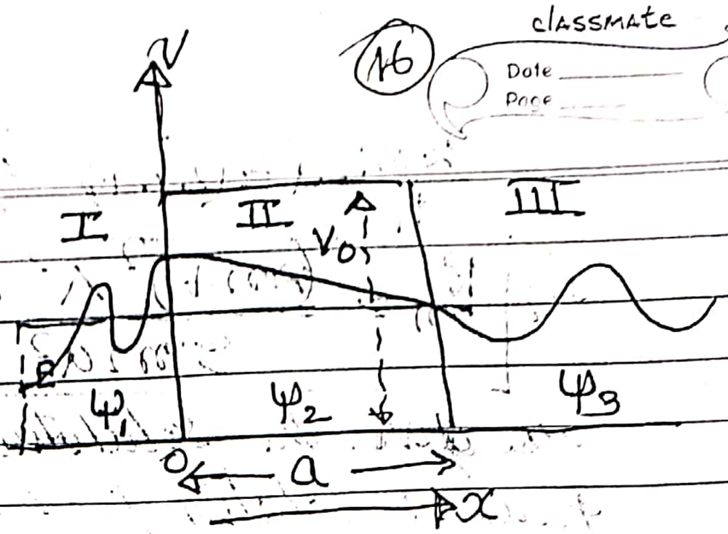
Discus the transmission of a particle through a rectangulal potential barrier. Discus briefly its application to the observed phenomena of α-decay at nuclei. Etc.

कणिका / Rectangular one dimensional potential barrier.

Let us consider a beam of particle of energy E incident from the left on a potential barrier of height V_0 and width a. The potential energy in region I & III is zero and that in region II is V_0.

Let ψ_1, ψ_2 & ψ_3 are the wave function in region I, II & III respectively.

The Schrödinger Equation in region I & (ii) & (iii) are give by



$$\frac{d^2\psi_1}{dx^2} + \frac{2mE}{\hbar^2}\psi_1 = 0 \quad (1)$$

$$\frac{d^2\psi_2}{dx^2} - \frac{2m}{\hbar^2}[E - V_0]\psi_2 = 0 \quad (2)$$

$$\frac{d^2\psi_3}{dx^2} + \frac{2m}{\hbar^2}[E]\psi_3 = 0 \quad (3)$$

Let $\frac{2mE}{\hbar^2} = K_0^2$ and $\frac{2m(V_0 - E)}{\hbar^2} = K^2$

$\therefore \frac{d^2\psi_1}{dx^2} + K_0^2\psi_1 = 0 \Rightarrow \psi_1 = Ae^{2K_0x} + Be^{-2K_0x}$ — (4)

$\frac{d^2\psi_2}{dx^2} - K\psi_2 = 0 \Rightarrow \psi_2 = Ce^{Kx} + De^{-Kx}$ — (5)

and $\frac{d^2\psi_3}{dx^2} + K_0^2\psi_3 = 0 \Rightarrow \psi_3 = Fe^{2K_0x} + Ge^{-2K_0x}$ — (6)

Where A = amplitude of intial wave, B = amplitude of reflected wave in region I, C = amplitude of penetrating wave in II, D = amplitude of reflected ~~transmitting~~ wave in region III and F = amplitude of transmitted wave in region III
G = amplitude of reflected wave in region III (nonexistance).

$\therefore \psi_3 = Fe^{2K_0x}$ — (7)

Applying boundary condition

(i) ψ & $\frac{d\psi}{dt}$ are continuous at $x=0$

$\therefore \psi_1 = \psi_2$ & $\frac{d\psi_1}{dt} = \frac{d\psi_2}{dt}$

From (4) & (5) $A + B = C + D$ —— (8)

and $iK_0 A - iK_0 B = CK + DK$ —— (9)

Adding and substracting (8) & (9) we get

$$A = \left(1 - \frac{iK}{K_0}\right)\frac{C}{2} + \left[1 + \frac{iK}{K_0}\right]D/2 \quad \text{—— (10)}$$

$$\& \; B = \left(1 + \frac{iK}{K_0}\right)\frac{C}{2} + \left(1 - \frac{iK}{K_0}\right)D/2 \quad \text{—— (11)}$$

(ii) Applying boundary condition at $x = a$

$\psi_2 = \psi_3$ & $\frac{d\psi_2}{dx} = \frac{d\psi_3}{dx}$

From (5) & (6)

$$Ce^{Ka} + De^{-Ka} = Fe^{iK_0 a} \quad \text{—— (12)}$$

$$\text{and } KCe^{Ka} - DKe^{-Ka} = iK_0 F e^{iK_0 a} \quad \text{—— (13)}$$

Evaluating (12) & (13)

$$C = \left(1 + \frac{iK_0}{K}\right)\frac{F}{2} e^{(iK_0 - K)a} \quad \text{—— (14)}$$

$$D = \left(1 - \frac{iK_0}{K}\right)\frac{F}{2} e^{(iK_0 + K)a} \quad \text{—— (15)}$$

If the barrier is thick Ka is very large. Hence to a first approximation C can be neglected

From eqn (10)

$$A = \left(1 + \frac{iK}{K_0}\right)\frac{D}{2}$$

$$= \left(1 + \frac{iK}{K_0}\right)\left(1 - \frac{iK_0}{K}\right)\frac{F}{4} e^{iK_0 a} \cdot e^{Ka} \quad \text{—— (16)}$$

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$\because Ka \gg K_0 a$

$$\therefore A = \left(1+\frac{2K}{K_0}\right)\left(1-\frac{2K_0}{K}\right)e^{Ka}\frac{F}{4}$$

$$\text{or, } \frac{A}{F} = \left(1+\frac{2K}{K_0}\right)\left(1-\frac{2K_0}{K}\right)\frac{e^{Ka}}{4}$$

$$\& \left(\frac{A}{F}\right)^* = \left(1-\frac{2K}{K_0}\right)\left(1+\frac{2K_0}{K}\right)\frac{e^{Ka}}{4}$$

$$\Rightarrow \left(\frac{A}{F}\right)\left(\frac{A}{F}\right)^* = \left(1+\frac{K^2}{K_0^2}\right)\left(1+\frac{K_0^2}{K^2}\right)\frac{e^{2Ka}}{16}$$

$$= \frac{(K_0^2+K^2)^2}{K_0^2 K^2}\cdot\frac{1}{16}e^{2Ka} \quad (17)$$

Substituting the value of K_0^2 & K^2 and solving

We get

$$\left(\frac{A}{F}\right)^2 = \left(\frac{A}{F}\right)\left(\frac{A}{F}\right)^* = \frac{V_0^2 e^{2Ka}}{16E(V_0-E)} \quad (18)$$

Result :- The ratio of intensity of the transmitted wave to the incident wave is k/a transmission coefficient T

$$T = \frac{F}{A} = \frac{16E(V_0-E)}{V_0^2 e^{2Ka}}$$

$$T = \frac{16E}{V_0}\left[1-\frac{E}{V_0}\right]e^{-2Ka} \quad (19)$$

This equation shows that the probability of penetrate the potential barrier is finite even $V_0 > E$. The α-particle emitted from the nucleus of Radium (Ra) has an energy $E = 4.88$ MeV while at the surface of the nucleus the potential energy of an α-particle is about 27.8 MeV.

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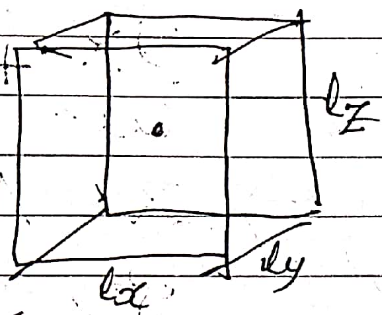
Discuss the transmission of a particle through a rectangular potential barrier. Discuss briefly its application to the observed Phenomena of α-decay

Obtain the Eigen function when a particle is kept in a rectangular box of dimension l_x, l_y, l_z. Find the eigen value of momentum and energy

S.E OF A PARTICLE IN A RECTANGULAR BOX

Let us consider a rectangular box of side l_x, l_y & l_z. A particle is free of mass m. The potential energy $V(x,y,z)$ of the particle inside the box is zero.



The time dependent Schrödinger wave equation is given by

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

$\because V = 0$ and $E = \frac{p^2}{2m}$

$$\therefore \nabla^2\psi + \frac{p^2}{\hbar^2}\psi = 0$$