Date:- 20/05/2020

Time:-10:00a.m. to 1:00p.m.

Chapter:- Hydrostatic

Topic:-

(i) Thrust on Plane surfaces

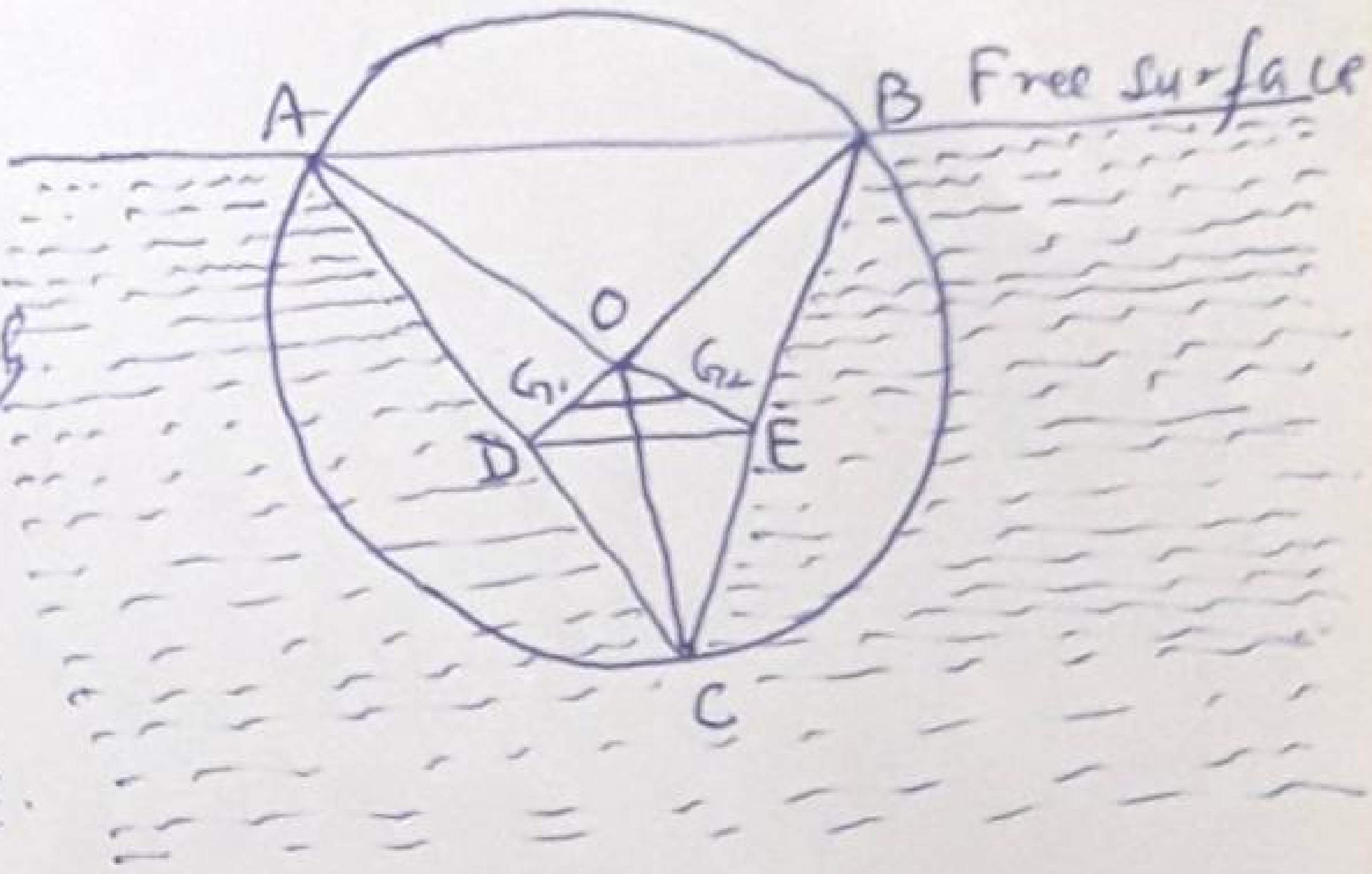
(ii) Centre of Pressure

By Professor (Dr.) Mohammad Eqbalu zafar

Thrust on Plane Surfaces

Ex-1 A triangle ABC is immersed in a liquid, its plane being vertical and the side AB in the surface; if O be the centre of the Circumscribed circle of $\triangle ABC$, prove that pressure on $\triangle OCA$: pressure on $\triangle OCB = \sin 2B : \sin 2A$

Soln: Let D and E be the mid-points of AC and BC respectively.



Let G_1 and G_2 be the centres of gravity of Δ's AOC and BOC respectively.

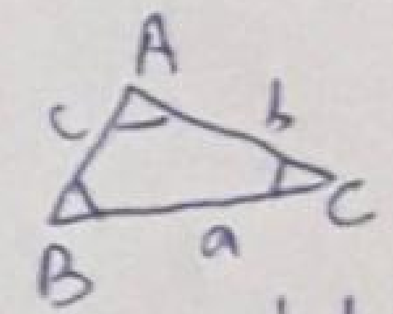
Then $OG_1 = \frac{2}{3}OD$ and $OG_2 = \frac{2}{3}OE$.

$\therefore \frac{OG_1}{OG_2} = \frac{OD}{OE}$.

Hence G_1G_2 is parallel to DE.

But DE is parallel to AB. $\therefore G_1G_2$ is parallel to AB.

$\therefore$ Depth of G_1 = Depth of $G_2 = \bar{z}$ (say)



$\therefore$ Area $= \frac{1}{2}bc\sin A$

$\therefore$ Whole pressure on $\Delta OCA = \rho g\bar{z} \cdot \frac{1}{2}OA \cdot OC \cdot \sin AOC$

$= \frac{1}{2}\rho g\bar{z}\, r^2 \sin 2B$

where r is the radius of the circumcircle, and $\angle AOC = 2B$.

Whole pressure on $\Delta OCB = \rho g\bar{z} \cdot \frac{1}{2}OC \cdot OB \cdot \sin BOC$

$= \frac{1}{2}\rho g\bar{z}\, r^2 \sin 2A$

Hence whole pressure on ΔOCA : whole pressure on ΔOCB

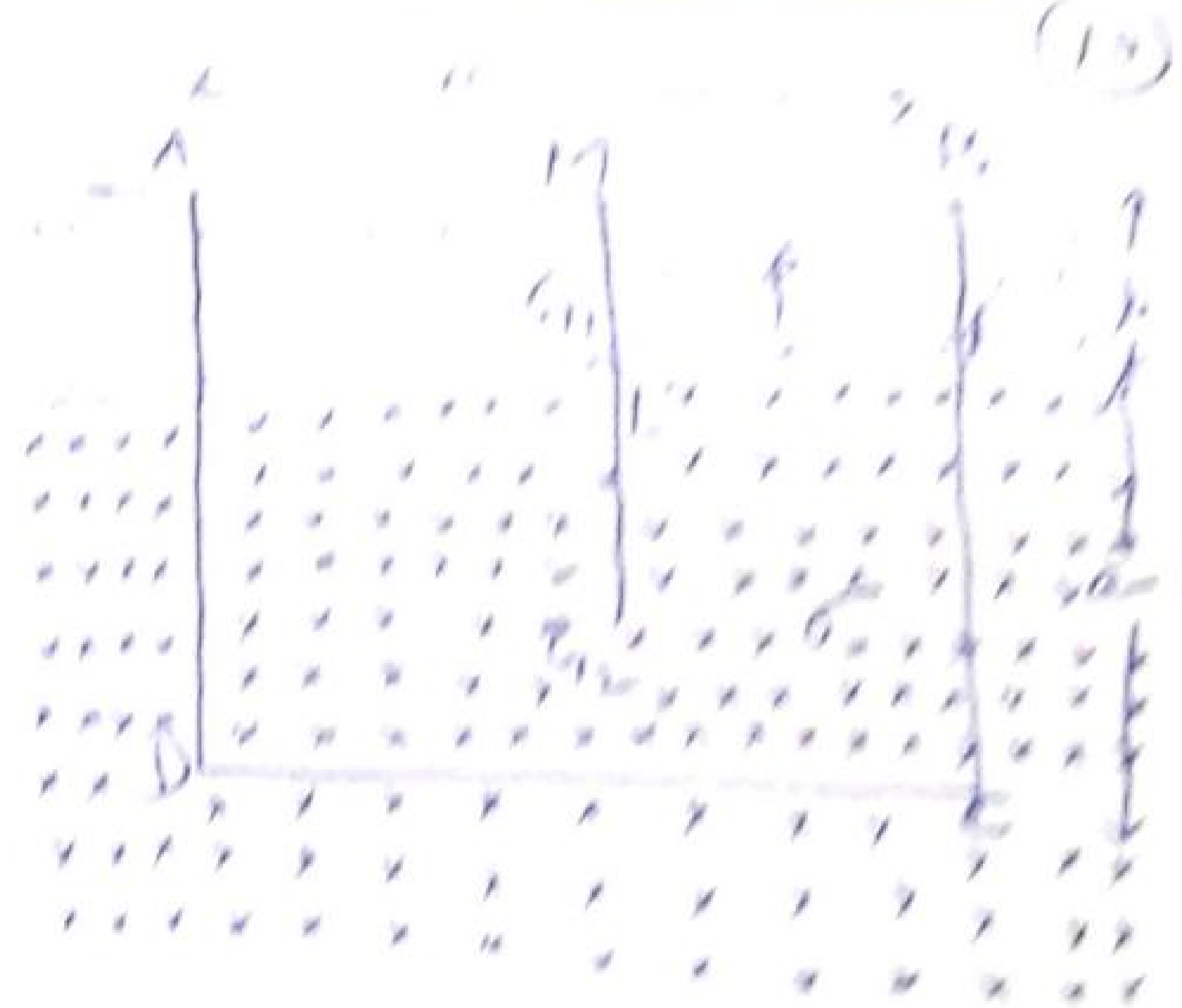
$= \sin 2B : \sin 2A$.

Ex-2 The lighter of two liquids of density ρ rests on the heavier of density ρ to a depth b. A square of side a is immersed in a vertical position with one side in the surface of the upper liquid. If the thrusts on the two portions of the square in contact with the two liquids be equal, prove that

$$b\rho(3b-2a) = \sigma(a-b)^2$$

Soln:- Let EF be the surface of separation of the two liquids. Let G_1 and G_2 be the centres of gravity of the two portions.

Now thrust on the portion ABFE $= \underset{\text{(formula)}}{\rho g\bar{z}S} = \rho g \cdot \frac{b}{2} \cdot ab$



and thrust on the portion EFCD

$= (\rho_1 h_1 + \rho_2 h_2)\, g\, S$ (formula)

i.e. (pressure at G_2) × area EFCD

$= \left(\rho \cdot b + \sigma\left(\frac{a-b}{2}\right)\right) g \cdot a(a-b)$

By question thrust on ABFE = thrust on EFCD

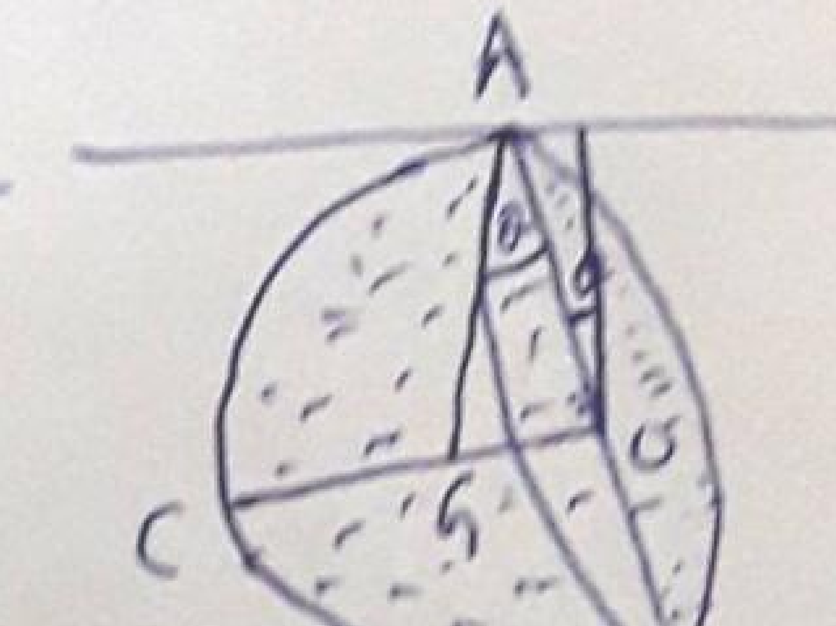
i.e. $\frac{1}{2}\rho g a b^2 = \left(\rho b + \sigma\left(\frac{a-b}{2}\right)\right) \cdot g \cdot a(a-b)$

i.e. $\rho b^2 = 2\rho b(a-b) + \sigma(a-b)^2$

i.e. $\rho b(b - 2a + 2b) = \sigma(a-b)^2$

Hence $b\rho(3b-2a) = \sigma(a-b)^2$

Ex-3 A hollow weightless hemisphere, filled with liquid is suspended freely from a point in the rim of its base; show that the thrus on the plane base is to the weight of the contained liquid as $12 : \sqrt{73}$.



Soln:- Let O be the centre of the base and G the C.G of the hemisphere full of liquid.

The $OG = \frac{3a}{8}$, where a is the radius of the hemisphere. The hemisphere full of liquid is in equilibrium under the action of the following forces.

<i> the weight of the liquid acting vertically downwards at G,

<ii> the force at the point of suspension A.

For equilibrium AG must be vertical. Let $\angle OAG = \theta$.

Then $\tan\theta = \frac{OG}{OA} = \frac{\frac{3}{8}a}{a} = \frac{3}{8}$ and $OL = OA\cos\theta = a\cos\theta$

Let w = weight per unit volume of the liquid.

Now thrust on the plane base $= w\bar{z}S$ (formula) $= w \cdot OL \cdot \pi a^2$

$= \pi w a\cos\theta \cdot a^2 = \pi w a^3 \cos\theta$.

The weight of the contained liquid $= \frac{2}{3}\pi a^3 w$.

$\therefore \dfrac{\text{The thrust on the plane base}}{\text{The weight of the contained liquid}} = \dfrac{3\cos\theta}{2} = \dfrac{3}{2\sec\theta}$

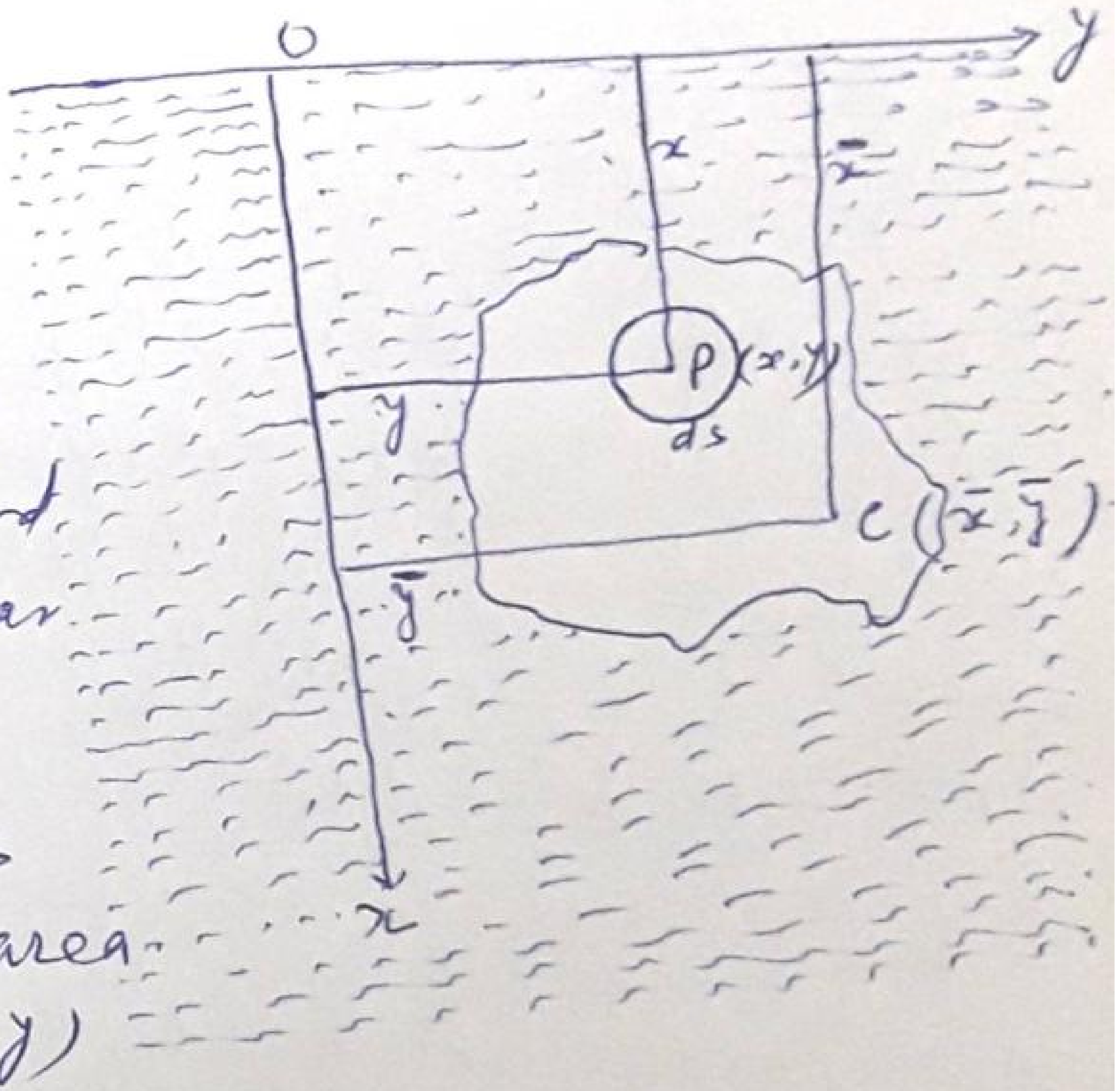
$= \frac{3}{2} \cdot \frac{1}{\sqrt{1+\tan^2\theta}} = \frac{3}{2} \cdot \frac{1}{\sqrt{1+(3/8)^2}} = \frac{12}{\sqrt{73}}$ Proved

Centre of Pressure (C.P)

Definition of the Centre of Pressure of a Plane Area.

The centre of pressure of a plane area immersed in a fluid is that point in the plane of the area at which the resultant thrust of the fluid on one side of plane area acts.

(Art) Find the co-ordinates of the centre of pressure of a plane area immersed in a liquid.



Sol:: Take the plane of the area as vertical.

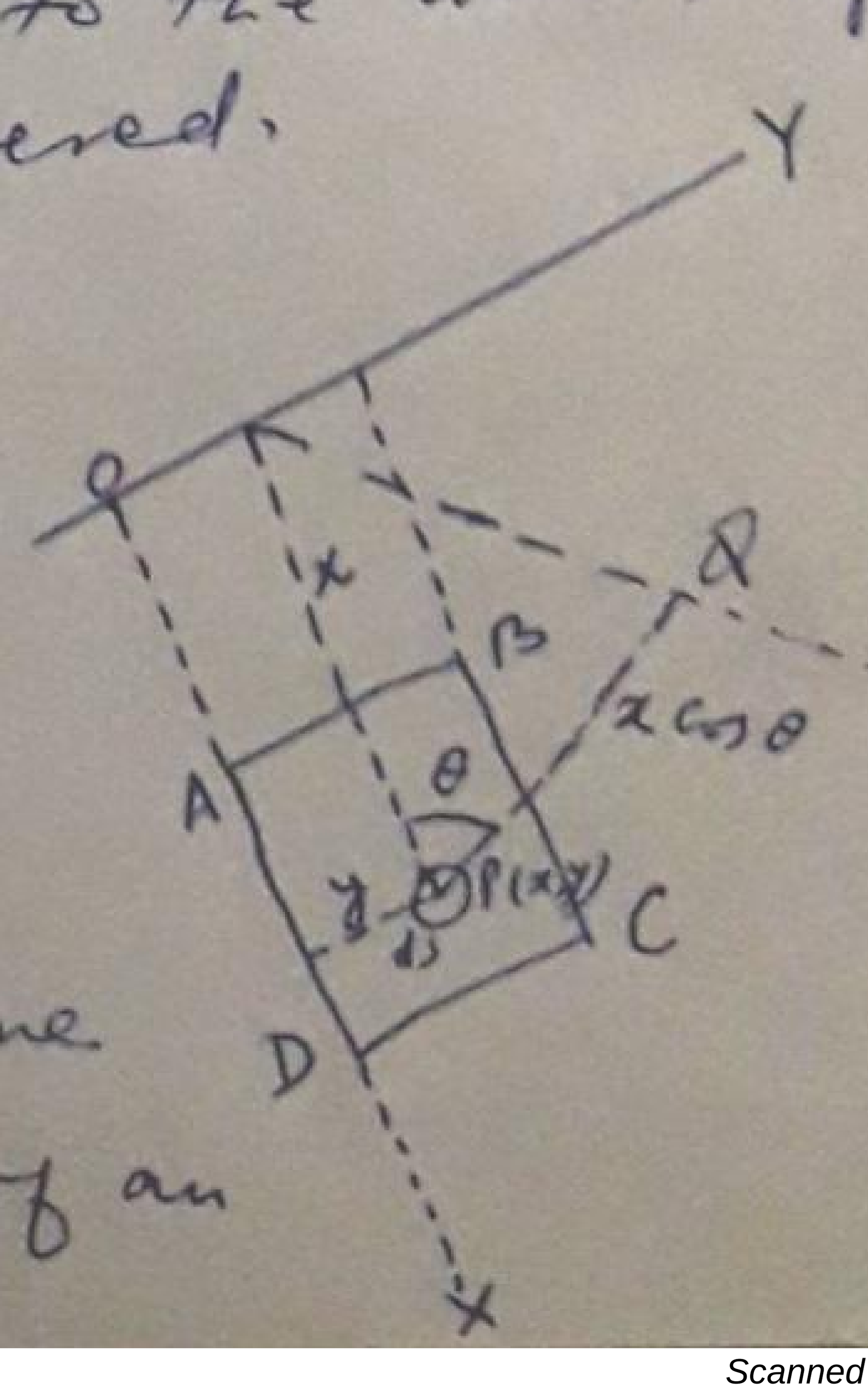
Let the intersection OY of the plane of the area with the free surface of the liquid be taken as the y-axis and a line ox in the plane perpendicular to OY as the x-axis.

Consider an element of area ds surrounding a point P(x,y) in the area. If p be the fluid pressure at (x,y) the thrust upon it is p ds.

If $(\bar{x},\bar{y})$ be the co-ordinates of the centre of pressure. If $(\bar{x},\bar{y})$ be the co-ordinates of the centre of parallel forces, we have by the theorem for the centre of parallel forces, we have

$$\bar{x} = \frac{\int xp\,ds}{\int p\,ds}, \quad \bar{y} = \frac{\int yp\,ds}{\int p\,ds}.$$

(Art) If the plane of an area immersed in a fluid be turned about its line of intersection with the effective surface, then the position, relative to the area, of the centre of pressure remains unaltered.



Sol: suppose ABCD is a plane area inclined at an angle θ to the vertical and let OY be its line of intersection with the effective surface.

Let us take OY as y-axis and a line OX perpendicular to OY and lying in the plane of the area as x-axis. Let ds be the area of an element surrounding P(x,y).

Let PQ be the perpendicular drawn from P to the effective surface.

Then $PQ = x\cos\theta$.

If p be the fluid pressure at P, then $p = \rho g \cdot PQ = \rho g x\cos\theta$, where ρ is the fluid density of the fluid

Let $(\bar{x}, \bar{y})$ be the Co-ordinates of the centre of pressure.

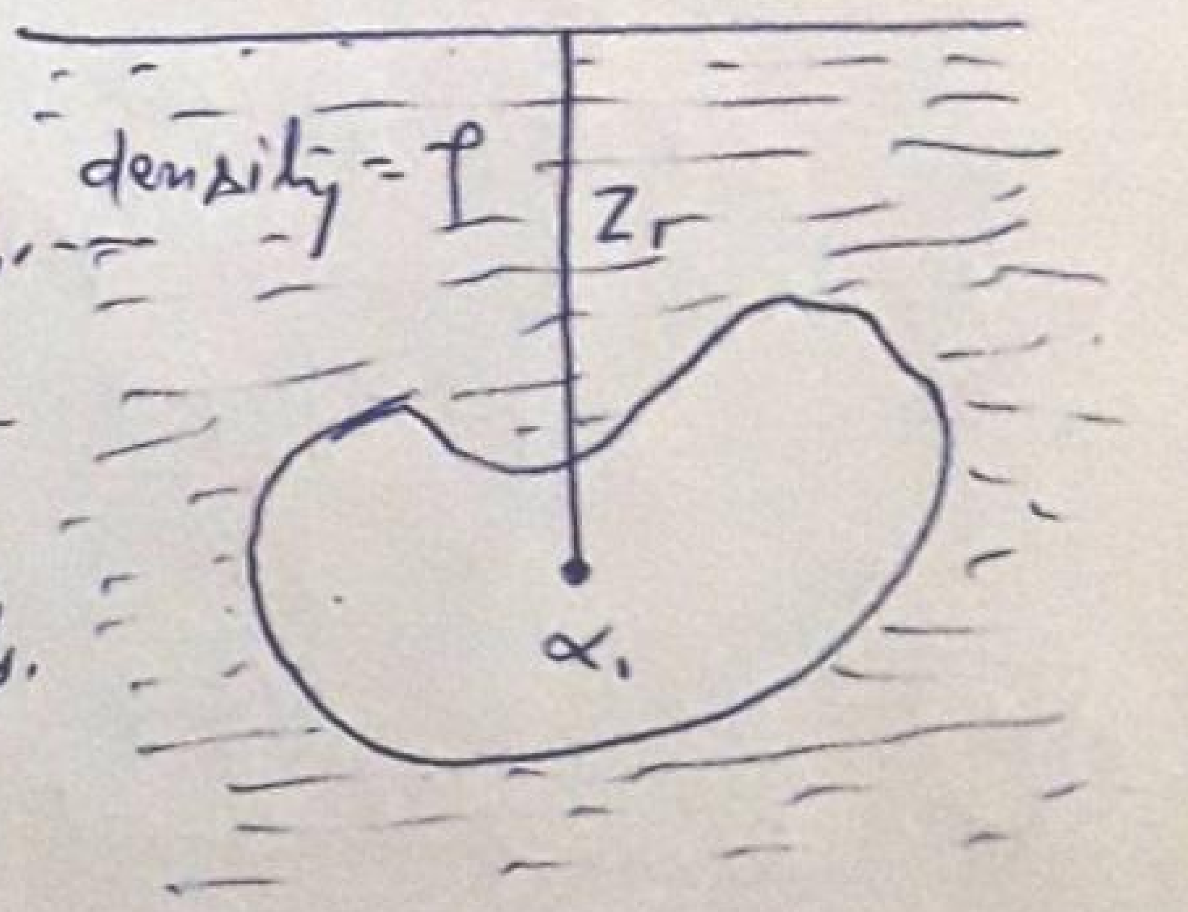
Then $$\bar{x} = \frac{\int xp\,ds}{\int p\,ds} = \frac{\int x\cdot \rho g x\cos\theta\,ds}{\int \rho g x\cos\theta\,ds} = \frac{\int x^2 ds}{\int x\,ds}$$

and $$\bar{y} = \frac{\int yp\,ds}{\int p\,ds} = \frac{\int y\cdot \rho g x\cos\theta\,ds}{\int \rho g x\cos\theta\,ds} = \frac{\int xy\,ds}{\int x\,ds}$$

The values of $\bar{x}$ and $\bar{y}$ donot involve θ. Hence the position of the centre of pressure relative to the area remains unaltered when θ changes.

Consequently there will be no loss of generality if we determine the centre of pressure by supposing the plane of the area to be vertical.

(Art) Prove that the depth of the centre of pressure always exceeds that of the centre of gravity of a plane area.



Soln:- Let the plane area be divided into a larger number of elements $\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_n, \ldots$ at depths $z_1, z_2, z_3, \ldots, z_n, \ldots$ beneath the effective surface.

Let z be measured positively downwards.

The thrust on α_1 is $w\alpha_1 z_1$.

Similarly the thrusts on element $\alpha_2, \alpha_3, \ldots, \alpha_n, \ldots$ are $w\alpha_2 z_2,\ w\alpha_3 z_3, \ldots w\alpha_n z_n, \ldots$

The depth of C.P $$= \frac{w\alpha_1 z_1\cdot z_1 + w\alpha_2 z_2\cdot z_2 + \cdots + w\alpha_n z_n\cdot z_n + \cdots}{w\alpha_1 z_1 + w\alpha_2 z_2 + \cdots + w\alpha_n z_n + \cdots}$$

$$= \frac{\alpha_1 z_1^2 + \alpha_2 z_2^2 + \cdots + \alpha_n z_n^2 + \cdots}{\alpha_1 z_1 + \alpha_2 z_2 + \cdots + \alpha_n z_n + \cdots}$$

The depth of C.G $$= \frac{w\alpha_1 z_1 + w\alpha_2 z_2 + \cdots + w\alpha_n z_n + \cdots}{w\alpha_1 + w\alpha_2 + \cdots + w\alpha_n + \cdots} = \frac{\alpha_1 z_1 + \alpha_2 z_2 + \cdots \alpha_n z_n + \cdots}{\alpha_1 + \alpha_2 + \alpha_3 + \cdots + \alpha_n + \cdots}$$

Now depth of C.P. − depth of C.G.

$$= \frac{\alpha_1 z_1^2 + \alpha_2 z_2^2 + \cdots + \alpha_n z_n^2 + \cdots}{\alpha_1 z_1 + \alpha_2 z_2 + \cdots + \alpha_n z_n + \cdots} - \frac{\alpha_1 z_1 + \alpha_2 z_2 + \cdots + \alpha_n z_n + \cdots}{\alpha_1 + \alpha_2 + \cdots + \alpha_n + \cdots}$$

$$= \frac{\alpha_1 \alpha_2 (z_1 - z_2)^2 + \alpha_1 \alpha_3 (z_1 - z_3)^2 + \cdots}{(\alpha_1 z_1 + \alpha_2 z_2 + \cdots + \alpha_n z_n + \cdots)(\alpha_1 + \alpha_2 + \cdots + \alpha_n + \cdots)}$$

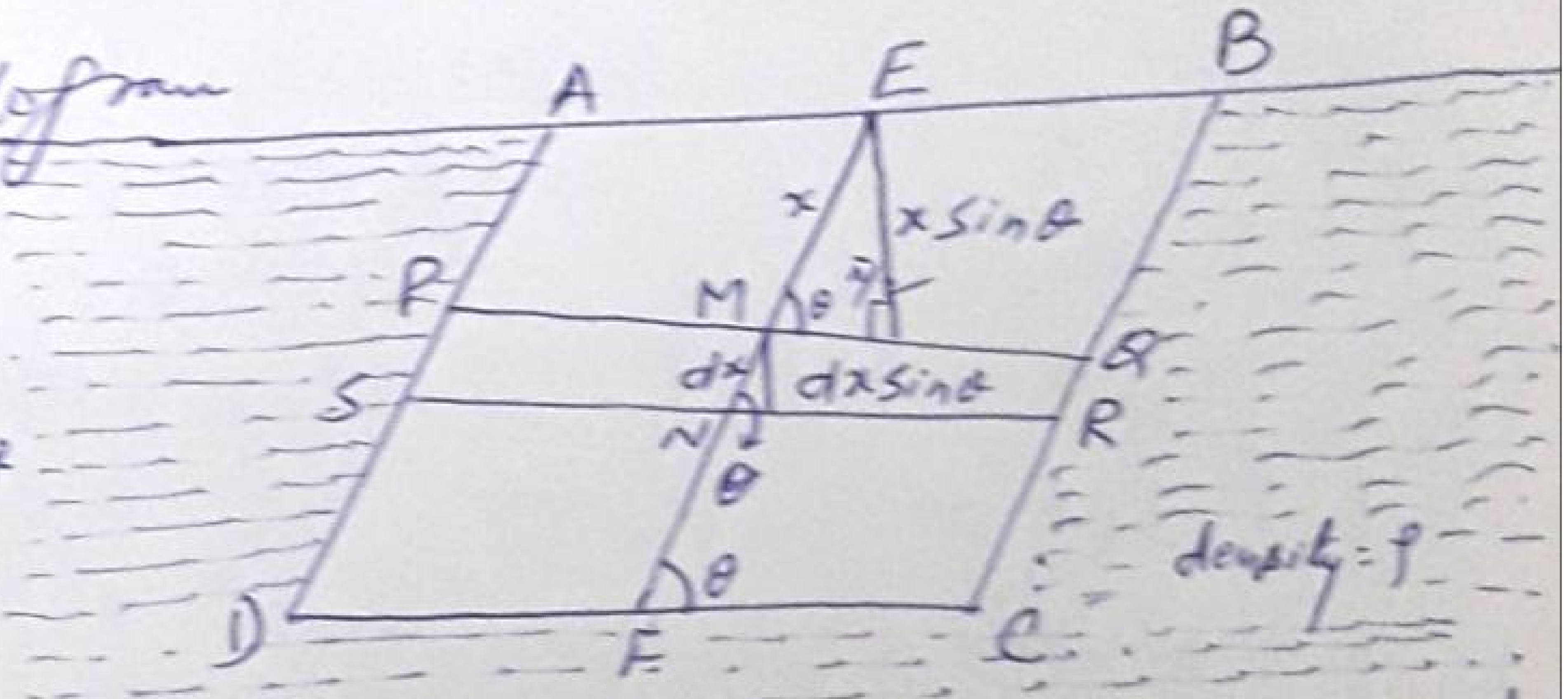
which is (+)ive.

Hence the depth of C.P. > the depth of C.G.

Note: If the plane area be horizontal, then $z_1 = z_2 = \cdots$

Hence the C.P. in this case will coincide with the C.G.

Ans Q Find the centre of pressure of a parallelogram immersed in a homogeneous liquid with one side in the free surface.



Soln:- Let ABCD be the parallelogram immersed in a liquid with side AB in the free surface.

If the parallelogram be not vertical, without loss of generality rotate it about AB so that it may be vertical.

Let EF be the line joining the middle points E and F of AB and CD respectively.

Let EF be inclined at an angle θ to the horizontal.

Suppose the parallelogram ABCD is divided into a large number of thin strips of the type PQRS || AB.

The thrusts on all such strips will act at their middle points which lie on EF.

Hence the centre of pressure of ||gm ABCD will lie on EF.

Let AB = a, AD = b, EM = x, MN = dx.

Then the depth of the strip PQRS from the free surface $= x\sin\theta$.

and the thickness of the strip $= dx\sin\theta$.

Area of the strip PQRS = PQ. $dx \sin\theta$
= $a \cdot dx \sin\theta = ds$ (say)

and the pressure at any point of the strip = $\rho g x \sin\theta$
= p (say)

where ρ is the density of the liquid.

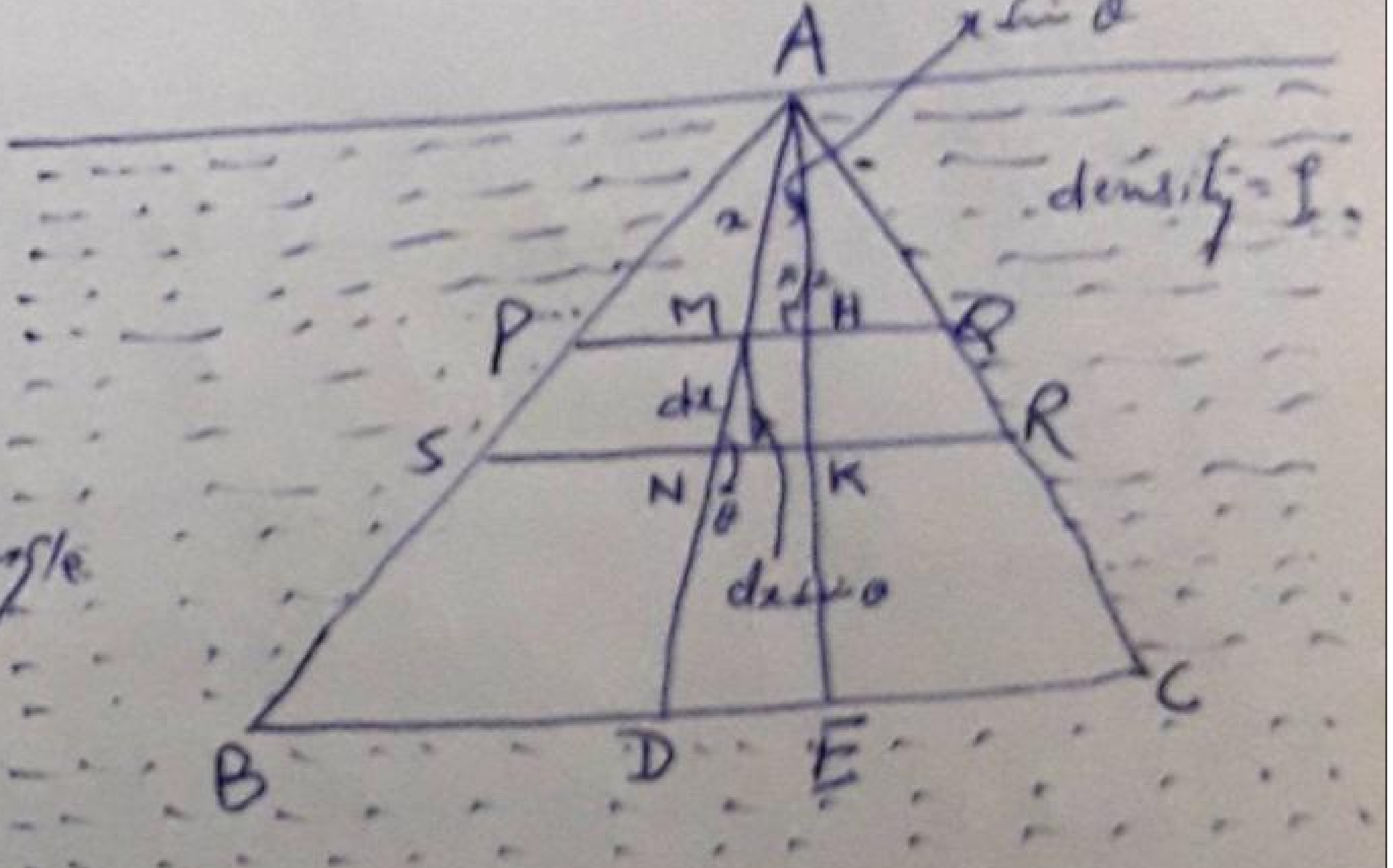
If $\bar{x}$ be the distance of C P of parallelogram ABCD from E along EF, then

$$\bar{x} = \frac{\int x p\, ds}{\int p\, ds} = \frac{\int_0^b x \cdot \rho g x \sin\theta \cdot a \sin\theta\, dx}{\int_0^b \rho g x \sin\theta \cdot a \sin\theta\, dx} = \frac{\int_0^b x^2 dx}{\int_0^b x\, dx} = \frac{\left[\frac{x^3}{3}\right]_0^b}{\left[\frac{x^2}{2}\right]_0^b}$$

$$= \frac{2}{3} b = \frac{2}{3} EF, \text{ since } EF = AD = b.$$

Note: In case of rectangle, $\theta = \frac{\pi}{2}$ so $\bar{x} = \frac{2}{3} EF$.

(Art) Find the Centre of pressure of a triangular area immersed in a homogeneous liquid with its vertex in the surface and base horizontal.



Soln:- Let ABC be the triangle with its vertex A in the free surface and base BC horizontal.
Let AD be the median of the triangle.
Divide the triangular area ABC into a large number of thin strips like PQRS parallel to BC.

The thrusts on all such strips will act at their middle points which lie on AD.
Hence the Centre of pressure of ΔABC will lie on AD.
Let AD make an angle θ with the horizon.
Let BC = a, AD = h, AM = x. MN = dx.
Then the depth of the strip from the free surface = AH = $x \sin\theta$
and the thickness of the strip = HK = $dx \sin\theta$.
Since the Δ^s APQ and ABC are similar, therefore $\frac{PQ}{BC} = \frac{AM}{AD}$

i.e. $\frac{PQ}{a} = \frac{x}{h}$

$\Rightarrow PQ = \frac{ax}{h}$.

Area of the strip PQRS $= PQ \cdot dx \sin\theta = \frac{ax}{h} \sin\theta \cdot dx$
$= ds$ (say)

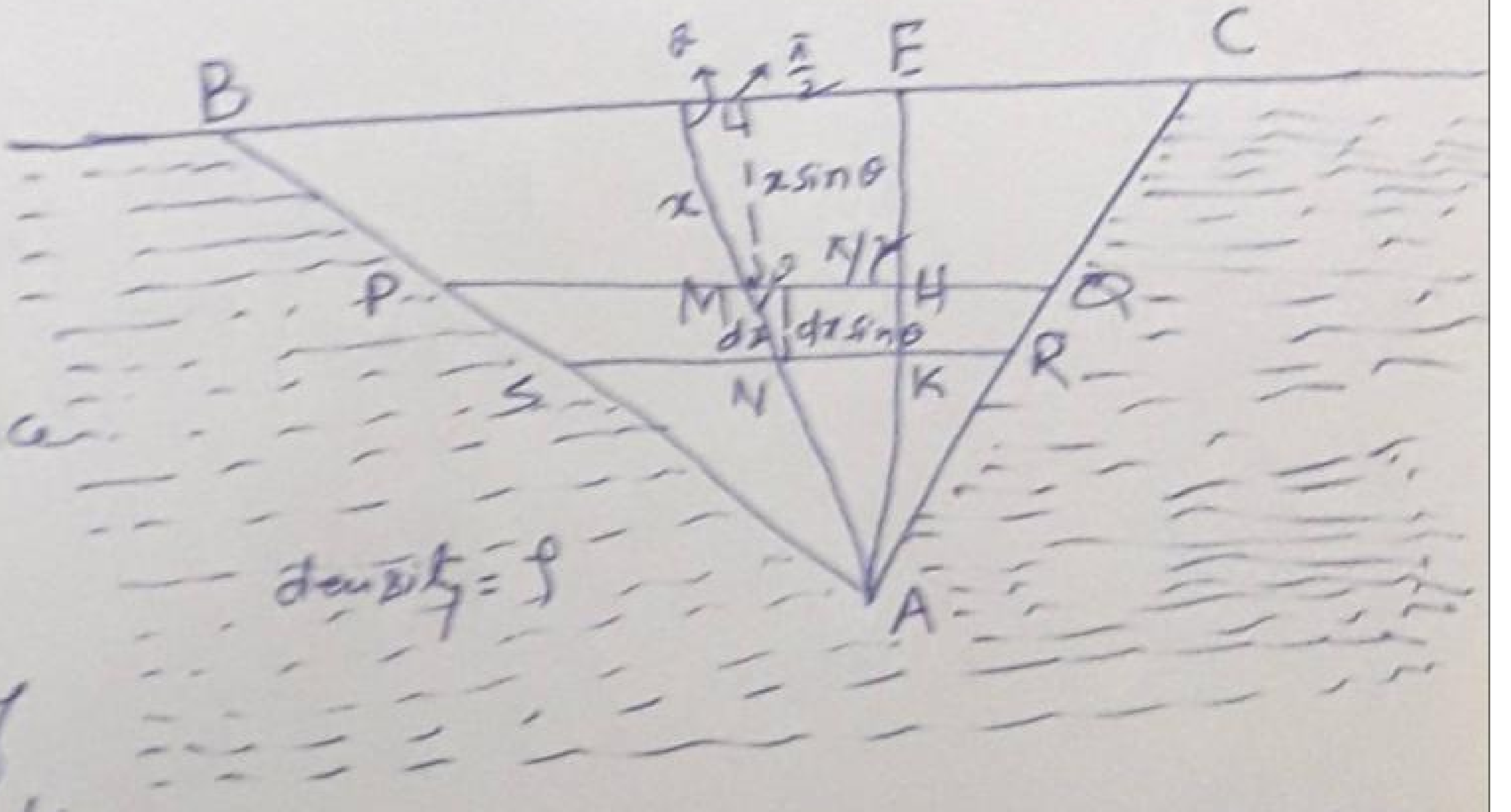
and the pressure at any of the strip $= \rho g x \sin\theta = p$ (say).
where ρ is the density of the liquid.

If $\bar{x}$ be the distance of the C.P of $\triangle ABC$ from A along AD
then
$$\bar{x} = \frac{\int x p\, ds}{\int p\, ds} = \frac{\int_0^h x \cdot \rho g x \sin\theta \cdot \frac{ax}{h} \sin\theta\, dx}{\int_0^h \rho g x \sin\theta \cdot \frac{ax}{h} \sin\theta\, dx}$$
$$= \frac{\int_0^h x^3 dx}{\int_0^h x^2 dx} = \frac{\left[\frac{x^4}{4}\right]_0^h}{\left[\frac{x^3}{3}\right]_0^h} = \frac{3}{4} h$$
$$= \frac{3}{4} AD$$

Q. (A.I) Find the centre of pressure of a triangular area immersed in a homogeneous liquid with one side in the surface of the liquid and vertex downward.

Soln:

Let ABC be the triangular area immersed in a liquid with side BC in the free surface and vertex A downwards.



Let AD be the median of $\triangle ABC$. Suppose the triangular area ABC is divided into a large number of thin strips of the type PQRS parallel to BC.

The thrusts on all such strips will act at their middle points which lie on AD.

Hence the centre of pressure of $\triangle ABC$ will lie on AD.

Let AD make an angle θ with BC.

Let $BC = a$, $AD = h$, $MD = x$, $MN = dx$.

Then the thickness of the strip $= HK = dx \sin\theta$ and the depth of PQ below the free surface $= EH = x \sin\theta$.

Since the $\triangle^s$ ABC and APQ are similar, therefore

$\frac{BC}{PQ} = \frac{AD}{AM}$, i.e. $\frac{a}{PQ} = \frac{h}{h-x} \Rightarrow PQ = \frac{a(h-x)}{h}$ (1b)

$\therefore$ Area of the strip PQRS $= PQ \cdot dx \sin\theta$

$$= \frac{a(h-x)}{h} \cdot dx \sin\theta = ds \text{ (say)}$$

and the pressure at any point of the strip $= \rho g x \sin\theta$
$= p$ (say).

ρ is the density of the liquid.

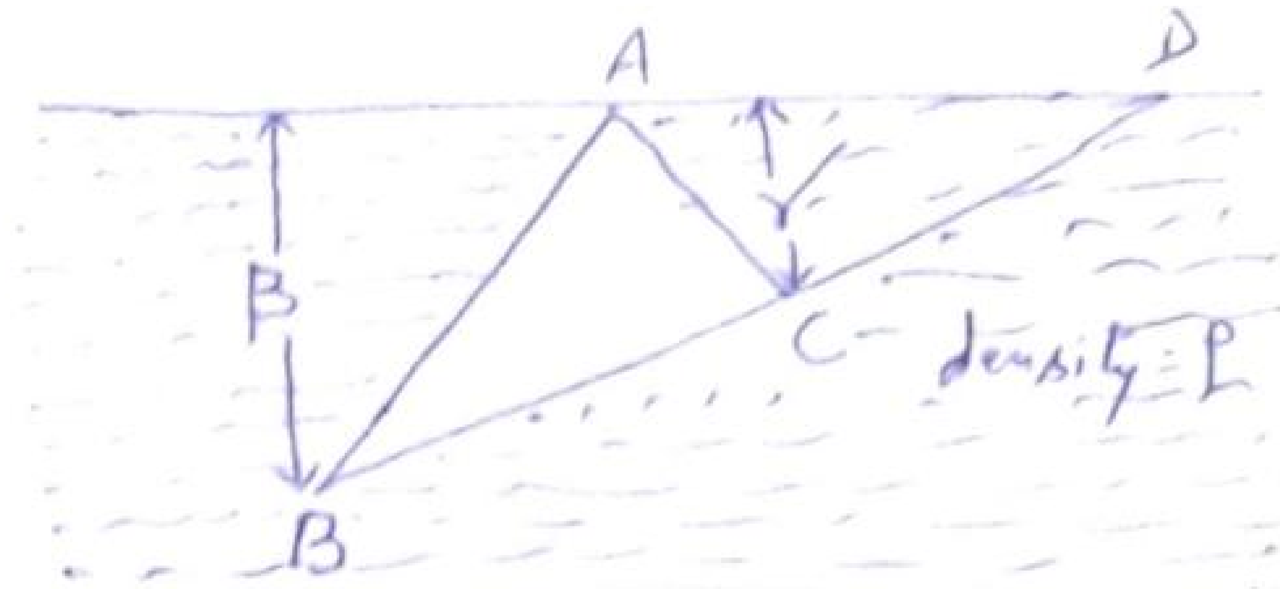
If $\bar{x}$ be the distance of the C.P of $\triangle ABC$ from D along DA, then

$$\bar{x} = \frac{\int x p \, ds}{\int p \, ds} = \frac{\int_0^h x \cdot \rho g x \sin\theta \, \frac{a(h-x)}{h} \sin\theta \, dx}{\int_0^h \rho g x \sin\theta \cdot \frac{a(h-x)}{h} \sin\theta \, dx}$$

$$= \frac{h\int_0^h x^2 dx - \int_0^h x^3 dx}{h\int_0^h x\,dx - \int_0^h x^2 dx} = \frac{h\left[\frac{x^3}{3}\right]_0^h - \left[\frac{x^4}{4}\right]_0^h}{h\left[\frac{x^2}{2}\right]_0^h - \left[\frac{x^3}{3}\right]_0^h}$$

$$= \frac{1}{2} \cdot \frac{4h^4 - 3h^4}{3h^3 - 2h^3} = \frac{1}{2} h = \frac{1}{2} AD.$$

Art. Find the depth of the centre of pressure of a triangle with a vertex in the free surface and the other two vertices at depth β and γ.



Soln:- Let ABC be the triangle immersed in a liquid with the vertex A in the free surface. Let the depths of B and C below the free surface be β and γ respectively.

Let BC be produced to meet the free surface in D.

Then $\Delta ABC = \Delta ABD - \Delta ACD$.

Let ρ be the density of the liquid.

Now thrust on $\Delta ABD = \left(\rho g \cdot \frac{1}{3}\beta\right)\cdot\left(\frac{1}{2}\cdot AD\cdot\beta\right) = \frac{1}{6}\rho g\beta^2\cdot AD = T_1$ (say)

thrust on $\Delta ACD = \left(\rho g\cdot\frac{1}{3}\gamma\right)\cdot\left(\frac{1}{2}\cdot AD\cdot\gamma\right) = \frac{1}{6}\rho g\gamma^2\cdot AD = T_2$ (say)

the depth of CP of $\Delta ABD = \frac{1}{2}\beta = z_1$ (say)

and the depth of CP of $\Delta ACD = \frac{1}{2}\gamma = z_2$ (say)

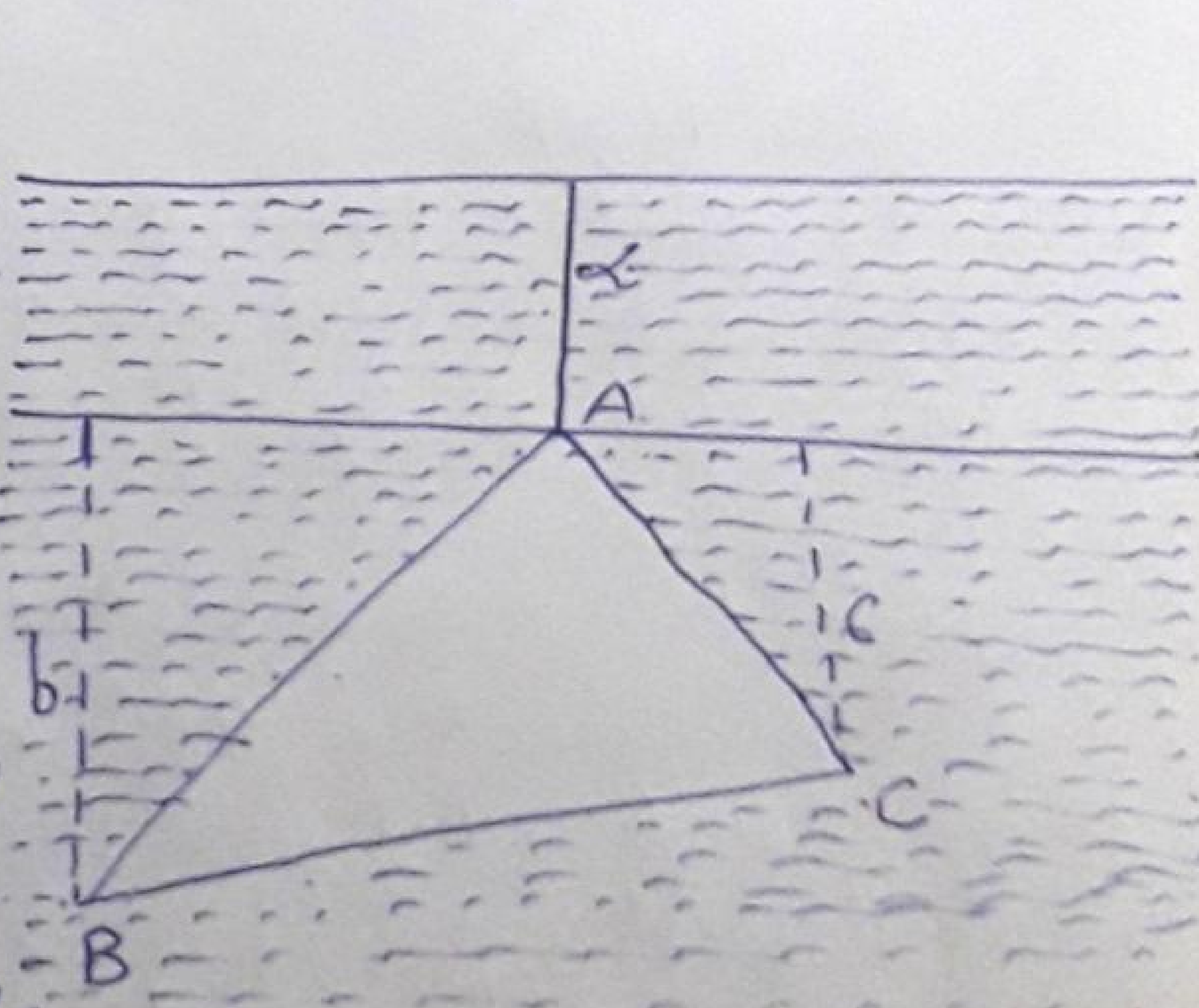
If z be the depth of CP. of ΔABC below the free surface.

then
$$z = \frac{T_1 z_1 - T_2 z_2}{T_1 - T_2} = \frac{\frac{1}{6}\rho g\beta^2\cdot AD\cdot\frac{1}{2}\beta - \frac{1}{6}\rho g\gamma^2\cdot AD\cdot\frac{1}{2}\gamma}{\frac{1}{6}\rho g\beta^2\cdot AD - \frac{1}{6}\rho g\gamma^2\cdot AD}$$
$$= \frac{1}{2}\,\frac{\beta^3-\gamma^3}{\beta^2-\gamma^2} = \frac{1}{2}\cdot\frac{\beta^2+\beta\gamma+\gamma^2}{\beta+\gamma}.$$

Art. Obtain the formula for the depth of the centre of pressure of a triangular area whose angular points are at depths α, β, γ.

Soln:

We know that the depth of the centre of pressure of a triangular lamina with a vertex in the free surface and the other two vertices at depths b and c is $\dfrac{b^2+bc+c^2}{2(b+c)}$



Now let the triangle be lowered through a distance α without rotation. Then the depths of B and C are $b+\alpha$ and $c+\alpha$.

$\therefore\ b+\alpha = \beta \Rightarrow b = \beta-\alpha$

$c+\alpha = \gamma \Rightarrow c = \gamma-\alpha$

On account of the lowering of the triangle through a depth α, there will be an additional thrust, $T_1 = w\Delta\alpha$, acting at the C.G. of the triangle whose depth below the horizontal through A is $z_1 = (b+c)/3$

The other force is the thrust $T_2 = w\Delta\frac{b+c}{3}$ acting at the C.P. whose depth below the horizontal through A is

$$z_2 = \frac{b^2+bc+c^2}{2(b+c)}.$$

∴ The depth of the new C.P. below the horizontal through A is

$$= \frac{T_1 z_1 + T_2 z_2}{T_1+T_2} = \frac{w\Delta\alpha\cdot\frac{b+c}{3} + w\Delta\frac{b+c}{3}\cdot\frac{b^2+bc+c^2}{2(b+c)}}{w\Delta\alpha + w\Delta\frac{b+c}{3}}$$

$$= \frac{b^2+bc+c^2+2\alpha(b+c)}{2(3\alpha+b+c)}$$

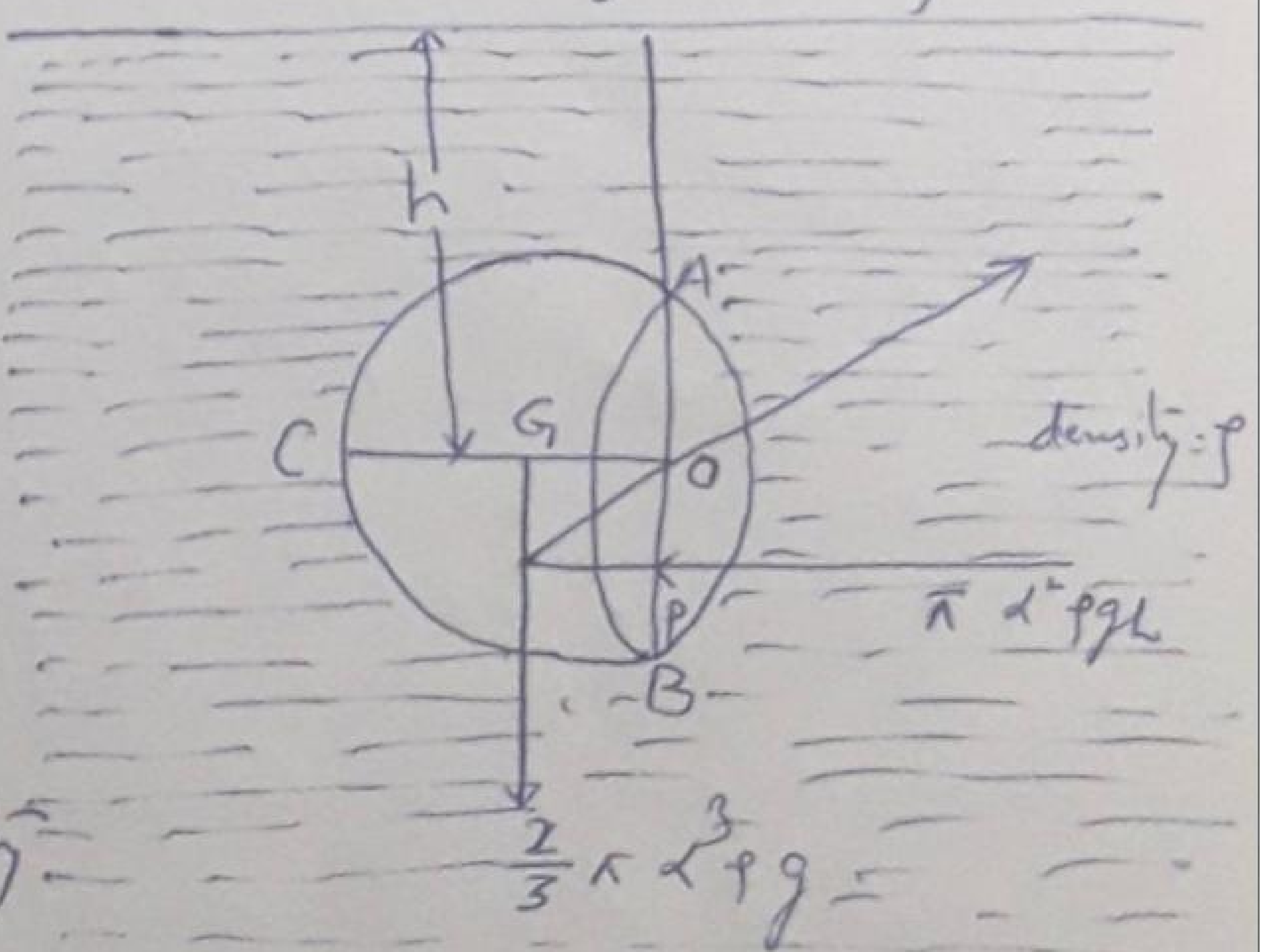
Hence the depth $\bar z$ of C.P. below the free surface is given by $\bar z = \alpha + \dfrac{b^2+bc+c^2+2\alpha(b+c)}{2(3\alpha+b+c)}$

$$= \alpha + \frac{(\beta-\alpha)^2+(\beta-\alpha)(\gamma-\alpha)+(\gamma-\alpha)^2+2\alpha(\beta-\alpha+\gamma-\alpha)}{2(3\alpha+\beta-\alpha+\gamma-\alpha)}$$

$$= \alpha + \frac{\beta^2-2\alpha\beta+\alpha^2+\beta\gamma-\alpha\gamma-\beta\alpha+\alpha\gamma+\gamma^2-2\gamma\alpha+\alpha^2+2\alpha\beta-4\alpha^2+2\alpha\gamma}{2(\alpha+\beta+\gamma)}$$

$$= \alpha + \frac{\beta^2+\gamma^2-\alpha^2-\alpha\beta+\beta\gamma-\gamma\alpha}{2(\alpha+\beta+\gamma)} = \frac{\alpha^2+\beta^2+\gamma^2+\alpha\beta+\beta\gamma+\gamma\alpha}{2(\alpha+\beta+\gamma)}.$$

(Art) Find the centre of pressure of a vertical circular area of radius a, wholly immersed in a homogeneous liquid with its centre at a depth h below the free surface.



Soln:- First Proof

Let us construct a hemi-sphere on the circle as base and consider the equilibrium of the liquid contained in it.

Suppose this area is vertical and its centre O is at a depth h from the free surface. Let ρ be the density of the liquid.

The forces acting on this liquid are:

(1) the weight of the liquid contained i.e. $\frac{2}{3}\pi a^3 \rho g$.

which is acting vertically downwards through G, the centre of gravity of the solid hemisphere, where $OG = \frac{3}{8}a$.

(ii) the thrust on the plane circular base $= \pi a^2 \rho g h$ which is acting at right angles to the base through P, the centre of Pressure of the face.

(iii) the resultant thrust on the curved surface, which must pass through O.

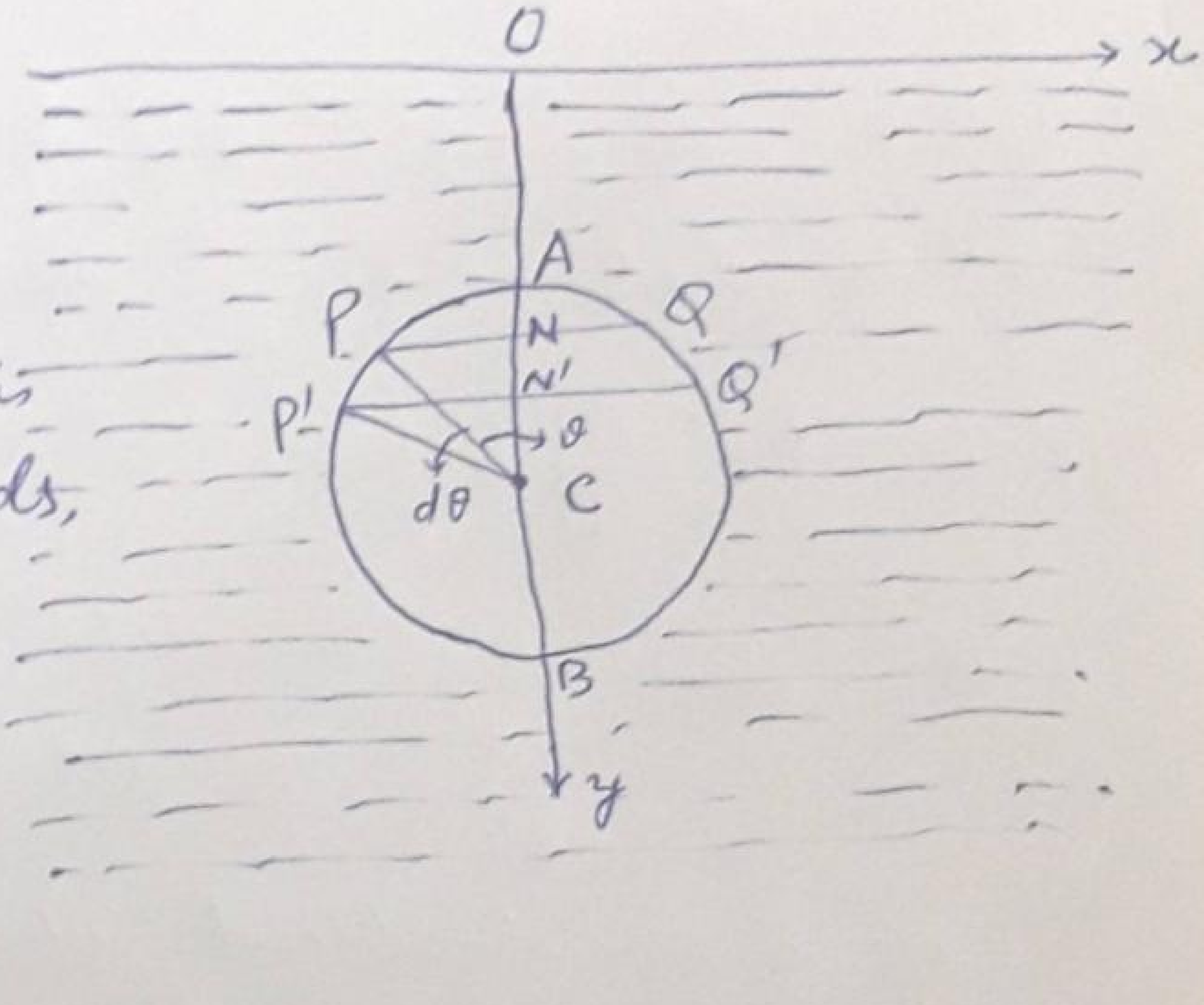
For equilibrium, taking moment about O,

$$\pi a^2 \rho g h \cdot OP = \frac{2}{3}\pi a^3 \rho g \cdot OG.$$

$$\therefore \quad OP = \frac{2}{3}\frac{a}{h}\cdot\frac{3}{8}a = \frac{a^2}{4h}.$$

Hence the depth of the C.P. from the surface of the liquid

$$= h + \frac{a^2}{4h}.$$



Second Proof:

Take the vertical line through the centre as axis of y. Since this line bisects all horizontal chords, therefore the centre of pressure must lie on this axis. Hence $\bar{x} = 0$ and we have to find $\bar{y}$ only.

Let $PQQ'P'$ be the elementary strip.

$ON = y$ and $NN' = dy$

Then $PQ = 2 \cdot PN = 2\sqrt{CP^2 - CN^2} = 2\sqrt{a^2 - (h-y)^2}$

Thrust on the strip $= g\rho y \cdot 2\sqrt{a^2-(h-y)^2}\,dy$.

and its moment about $Ox = \rho g y^2 \cdot 2\sqrt{a^2-(h-y)^2}\,dy$.

$$\therefore \quad \bar{y}\int_{h-a}^{h+a} 2g\rho y\sqrt{a^2-(h-y)^2}\,dy = \int_{h-a}^{h+a} 2g\rho y^2\sqrt{a^2-(h-y)^2}\,dy$$

$$\text{i.e.} \quad \bar{y}\int_0^{\pi} 2g\rho a^2\sin^2\theta\,(h-a\cos\theta)\,d\theta = \int_0^{\pi} 2g\rho a^2\sin^2\theta\,(h-a\cos\theta)^2\,d\theta$$

where $\angle ACP = \theta$, $\angle PCP' = d\theta$, $h - y = a\cos\theta$.

or $\bar{y}\left[h\int_0^{\pi}\frac{1-\cos 2\theta}{2}\,d\theta - a\int_0^{\pi}\sin^2\theta\, d(\sin\theta)\right]$

$$= h^2\int_0^{\pi}\frac{1-\cos 2\theta}{2}\,d\theta - 2ah\int_0^{\pi}\sin^2\theta\, d(\sin\theta) + \frac{a^2}{4}\int_0^{\pi}\frac{1-\cos 4\theta}{2}\,d\theta$$

or $\bar{y}\left[\frac{h}{2}\left(\theta-\frac{1}{2}\sin 2\theta\right) - \frac{a}{3}\sin^3\theta\right]_0^{\pi} = \frac{h^2}{2}\left[\theta-\frac{1}{2}\sin 2\theta\right]_0^{\pi} -$

$$\frac{2ah}{3}\left[\sin^3\theta\right]_0^{\pi} + \frac{a^2}{8}\left[\theta-\frac{1}{4}\sin 4\theta\right]_0^{\pi}$$

i.e. $\frac{\pi h}{2}\cdot\bar{y} = \frac{\pi h^2}{2} + \frac{\pi a^2}{8}$ i.e. $\bar{y} = h + \frac{a^2}{4h}$.

Hence the Centre of pressure of a Circle is at a distance $\frac{a^2}{4h}$ below the centre.

i.e. $h + \frac{a^2}{4h}$ below the free surface.

Note: Here h = the depth of the centre of the circle;

$\frac{a^2}{4h}$ = the distance between its centre and C.P

If h is doubled, then the distance between the centre and the C.P. $= \frac{a^2}{8h} = \frac{1}{2}\cdot\frac{a^2}{4h} = \frac{1}{2}\times$ the original distance between the Centre and the C.P.

Hence we arrive at a result. ie