Date - 04/05/2020

Time - 10:00 to 11:30

DEGREE - 1 (H + S)

SOLID GEOMETRY (3 D)

CHAPTER - EQUATION OF A PLANE

BY PROFESSOR
DR. MOHAMMAD EQBALU ZAFAR

Scanned by TapScanner

Solid Geometry (3D)

Chapter Equation of a Plane

Problem-1 Find the equation of the plane which
(i) cuts off intercepts a, b, c from the axes.
(ii) passes through the points $A(0,1,1)$, $B(1,1,2)$ and $C(-1,2,-2)$

Soln:- (i) We know that the equation of the plane in intercept form $\alpha x + \beta y + \gamma z + \delta = 0$ — (1)

The plane cuts the axes at A, B, C s.t. $\left.\begin{array}{l} OA = a \\ OB = b \\ OC = c \end{array}\right\}$

i.e. (1) passes through the points $A(a,0,0)$, $B(0,b,0)$ and $C(0,0,c)$.

So $\left.\begin{array}{l} a\alpha + \delta = 0 \Rightarrow \alpha = -\delta/a \\ b\beta + \delta = 0 \Rightarrow \beta = -\delta/b \\ c\gamma + \delta = 0 \Rightarrow \gamma = -\delta/c \end{array}\right\}$ substituting these values in eqn (1), we get

$$\left(-\frac{\delta}{a}\right)x + \left(-\frac{\delta}{b}\right)y + \left(-\frac{\delta}{c}\right)z + \delta = 0$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

(ii) We know that equation of the plane in three points form (x_1, y_1, z_1)

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \quad — (1)$$

Any plane through $(0,1,1)$ is $a(x-0) + b(y-1) + c(z-1) = 0$ — (2)

It will pass through $(1,1,2)$ and $(-1,2,-2)$

if $a(1-0) + b(1-1) + c(2-1) = 0 \Rightarrow a + c = 0$

and $a(-1-0) + b(2-1) + c(-2-1) = 0 \Rightarrow -a + b - 3c = 0$

$$\frac{a}{-1} = \frac{b}{2} = \frac{c}{1}$$ solving the above two eqns (cross-multiply)

Putting these values in (2) $-1 \cdot x + 2(y-1) + 1(z-1) = 0$

$\Rightarrow x - 2y - z + 3 = 0$ is the required eqn of the plane.

Scanned by TapScanner

Problem-2 Find the eqn of the plane which passes through the point $(3,-3,1)$ and is

(i) parallel to the plane $2x+3y+5z+6=0$

(ii) normal to the line joining the points $(3,2,-1)$ and $(2,-1,5)$.

(iii) perpendicular to the planes $7x+y+2z=6$ and $3x+5y-6z=8$.

Soln:- (1) We know that any plane parallel to the plane $ax+by+cz+d=0$ is $ax+by+cz+k=0$

So any plane parallel to the given plane is $2x+3y+5z+K=0$ which goes through $(3,-3,1)$ if $K=-2$

Thus the required plane is $2x+3y+5z=2$

(ii) Any plane through (x_1,y_1,z_1) is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

i.e. through $(3,-3,1)$ is $a(x-3)+b(y+3)+c(z-1)=0$ (1)

The d.cs of the line joining the points $(3,2,-1)$ and $(2,-1,5)$ are proportional to $1,3,-6$.

This line is normal to the plane (1)

$\therefore$ a, b, c are proportional to $1,3,-6$

From (1) $1(x-3)+3(y+3)-6(z-1)=0$

$\Rightarrow x+3y-6z+12=0$

(iii) Any plane through $(3,-3,1)$ is

$a(x-3)+b(y+3)+c(z-1)=0$ which will be perpendicular to the planes

$7x+y+2z=6$ and $3x+5y-6z=8$

$7a+b+2c=0$ and $3a+5b-6c=0$

Formula

The Planes $ax+by+cz+d=0$ is $\perp$ to $a'x+b'y+c'z+d'=0$ if $aa'+bb'+cc'=0$

Solving by cross-multiplication

$\frac{a}{1}=\frac{b}{-3}=\frac{c}{-2}$

$\therefore$ Required eqn is $1(x-3)-3(y+3)-2(z-1)=0$

$\Rightarrow x-3y-2z-10=0$ Ans

Problem-3 The plane $4x+5y-z=7$ is rotated through a right angle about its line of intersection with the plane $2x+3y-3z=5$. Find the eqn of this plane in its new position.

Soln: Any plane through the line of intersection of

$$4x+5y-z=7 \quad (1) \quad \text{and} \quad 2x+3y-3z=5 \quad (2)$$

is $(4+2k)x+(5+3k)y-(1+3k)z-(7+5k)=0 \quad (3)$

The new position of (1) when rotated through a rt. angle, is such that (1) and (3) are perpendicular.
This requires that $4(4+2k)+5(5+3k)+(1+3k)=0$

$$\Rightarrow 26k+42=0 \Rightarrow k=-21/13$$

Put the value of k in (3) $10x+2y+50z+14=0$

$$\Rightarrow 5x+y+25z+7=0$$ is the required plane.

Problem-4 Find the distance between the parallel lines $2x-2y+z+3=0$ and $4x-4y+2z+9=0$. Find also the eqn of the parallel plane that lies mid-way between the given planes.

Soln: The distance between the given planes is the perpendicular distance of any point on one of the planes from the other.

A point on the first plane is $(0,0,-3)$.

Required distance = $\perp$ distance of $(0,0,-3)$ from $4x-4y+2z+9=0$ $= \dfrac{-6+9}{\sqrt{16+16+4}} = \dfrac{3}{6} = \dfrac{1}{2}$

Let the equation of the parallel plane that lies mid-way between the given planes be $2x-2y+z+k=0 \quad (1)$

Now distance of any point $(0,0,-3)$ on the first plane from (1) should be $1/4$

$$\therefore \pm \frac{-3+K}{\sqrt{4+4+1}} = \frac{1}{4} \Rightarrow K = \frac{15}{4} \text{ or } \frac{9}{4}$$

Thus the required plane is $2x - 2y + z + \frac{15}{4} = 0$

Assume that $K = \frac{15}{4}$ and verify that the distance of a point on this plane $4x - 4y + 2z + 9 = 0$ is also $\frac{1}{4}$.

A point on this plane is $(0, 0, -\frac{9}{4})$.

Its distance from the plane ① $= \dfrac{-9/2 + 15/4}{3} = \dfrac{1}{4}$ (in magnitude)

Thus $K = \frac{9}{4}$ is not admissible.

$\therefore$ The required plane is $2x - 2y + z + \frac{15}{4} = 0$.

Problem 5 A variable plane is at a constant distance p from the origin and meets the axes at A, B, C. Find the locus of the centroid of the tetrahedron OABC.

Soln: As the given plane is at a $\perp$ distance p from the origin, therefore its eqn is of the form

$$lx + my + nz = p \quad \text{—} ①$$

where l, m, n are the d.c's of the $\perp$ from the origin.

① may be written as $\dfrac{x}{(p/l)} + \dfrac{y}{(p/m)} + \dfrac{z}{(p/n)} = 1.$

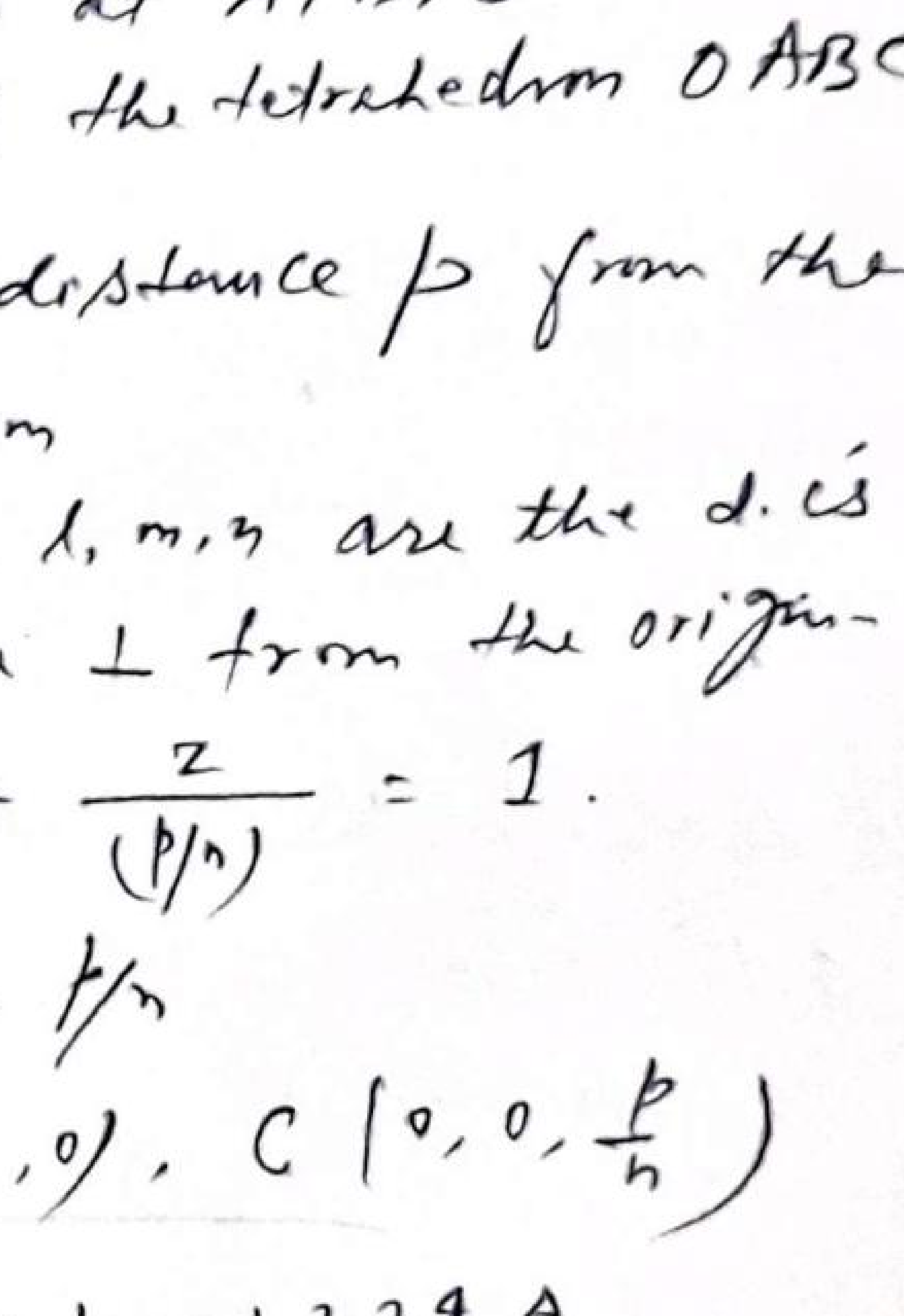
So that $OA = p/l$, $OB = p/m$, $OC = p/n$

$\therefore A = \left(\frac{p}{l}, 0, 0\right)$, $B = \left(0, \frac{p}{m}, 0\right)$, $C\left(0, 0, \frac{p}{n}\right)$

Formula

centroid of tetrahedron vertices are (x_r, y_r, z_r), $r = 1, 2, 3, 4$

$$\left[\frac{\sum x_r}{4}, \frac{\sum y_r}{4}, \frac{\sum z_r}{4}\right]$$



Thus the co-ordinates of the centroid of tetrahedron OABC are $\left(\frac{p}{4l}, \frac{p}{4m}, \frac{p}{4n}\right)$

$$\Rightarrow \frac{1}{x_1^2} + \frac{1}{y_1^2} + \frac{1}{z_1^2} = \frac{16}{p^2}(l^2 + m^2 + n^2) = \frac{16}{p^2} \qquad [\because l^2 + m^2 + n^2 = 1]$$

Thus the locus of G are $x^{-2} + y^{-2} + z^{-2} = 16p^{-2}$ Ans

Problem-6 A variable plane at a constant distance p from the origin meets the axes in A, B, C. Planes are drawn through A, B, C parallel to the Co-ordinate planes, Show that the locus of their point of intersection is given by $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.

Soln: Let the variable plane be $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

Its distance from the origin $= \frac{1}{\sqrt{a^{-2} + b^{-2} + c^{-2}}} = p$ (given)

i.e. $a^{-2} + b^{-2} + c^{-2} = p^{-2}$ —— (1)

Since $OA = a$, $OB = b$, $OC = c$

so ~~eqns~~ of the planes through A, B, C parallel to yz, zx and xy planes are $x = a$, $y = b$, $z = c$.

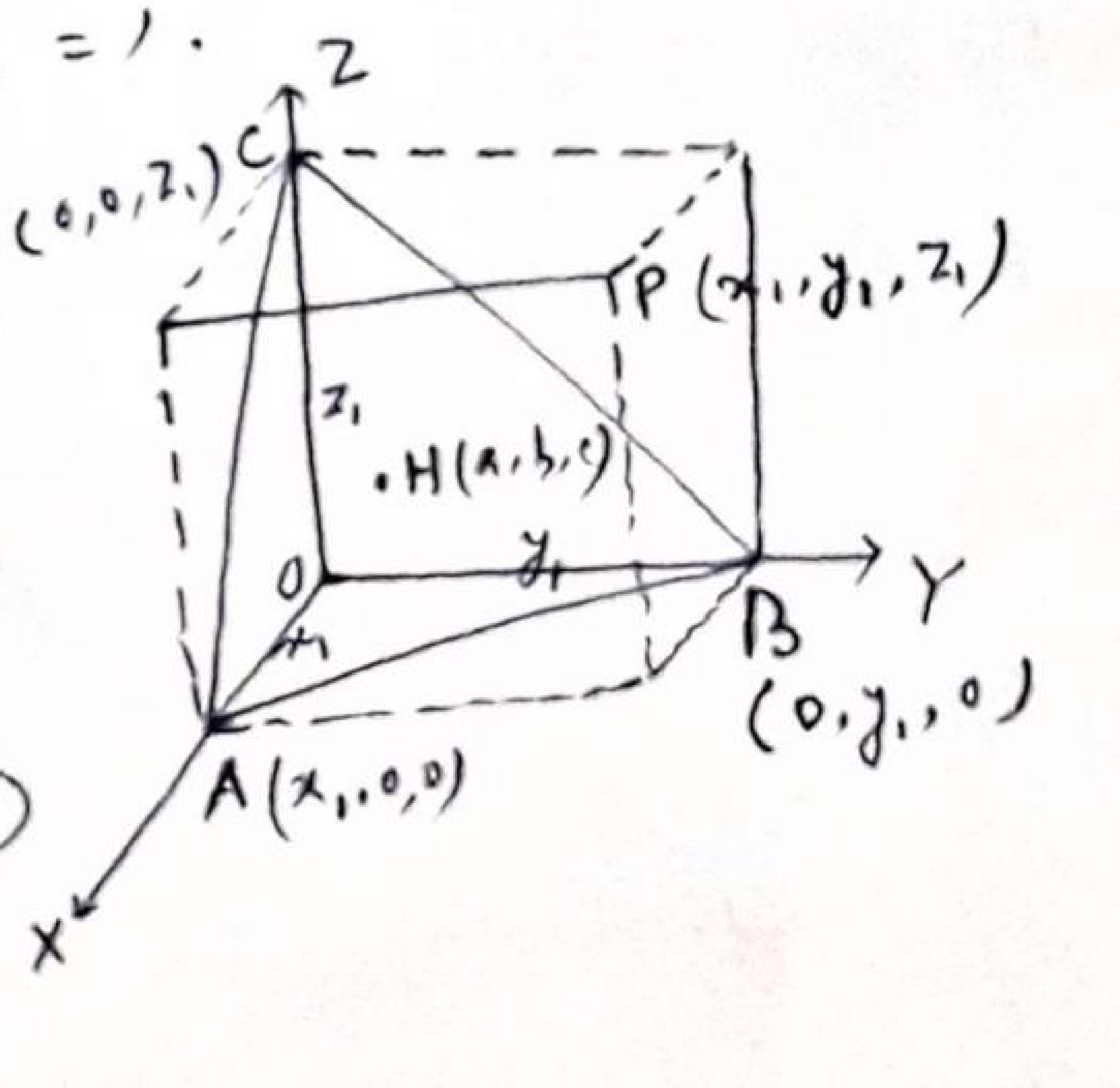
Let the point of intersection of these three planes be (x_1, y_1, z_1). Then $x_1 = a$, $y_1 = b$, $z_1 = c$ —— (2)

Putting the value from (2) in (1)

$$x_1^{-2} + y_1^{-2} + z_1^{-2} = p^{-2}$$

Thus the locus of (x_1, y_1, z_1) is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.

Problem-7 A variable plane passes through the fixed point (a, b, c) and meets the Co-ordinate axes in A, B, C. Show that the locus of the point common to the planes through A, B, C parallel to the Co-ordinate planes is $a/x + b/y + c/z = 1$.



Soln: Let ABC be any plane through the fixed point $H(a, b, c)$

s.t. $OA = x_1$, $OB = y_1$, $OC = z_1$.

Then its eqn is $\frac{x}{x_1} + \frac{y}{y_1} + \frac{z}{z_1} = 1$ —— (1)

$\boxed{\because \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1}$

Scanned by TapScanner

The planes through A, B, C parallel to the co-ordinate planes are $x = x_1$, $y = y_1$, $z = z_1$ which meet in $P(x_1, y_1, z_1)$.

Thus changing x_1 to x, y_1 to y and z_1 to z in the locus of the P is $a/x + b/y + c/z = 1$.

Problem-8 Find the eqns to the two planes which bisect the angles between the planes $3x - 4y + 5z = 3$, $5x + 3y - 4z = 9$.

Also point out which of the planes bisects the acute angle.

Sols: ~~Also point out which of the planes bisects the acute angle.~~

The eqns of the planes bisecting the angles between the given planes are

$$\frac{3x - 4y + 5z - 3}{\sqrt{(3)^2 + (-4)^2 + (5)^2}} = \pm \frac{5x + 3y - 4z - 9}{\sqrt{(5)^2 + (3)^2 + (-4)^2}}$$

$$\Rightarrow \quad 2x + 7y - 9z - 6 = 0 \quad \text{—} \; (1)$$

$$8x - y + z - 12 = 0 \quad \text{—} \; (2)$$

which are the required planes.

Let θ be the angle between (1) and ~~(2)~~ either of the given planes

$$5x + 3y - 4z = 9$$

Then,
$$\cos\theta = \frac{2 \times 5 + 7 \times 3 + (-9) \times (-4)}{\sqrt{(2)^2 + (7)^2 + (-9)^2} \sqrt{(5)^2 + (3)^2 + (-4)^2}}$$

27
10
31
67

$$= \frac{67}{5\sqrt{268}}$$

$$\tan\theta = \frac{\sqrt{2211}}{67} < 1 \quad \Rightarrow \theta < 45^\circ.$$

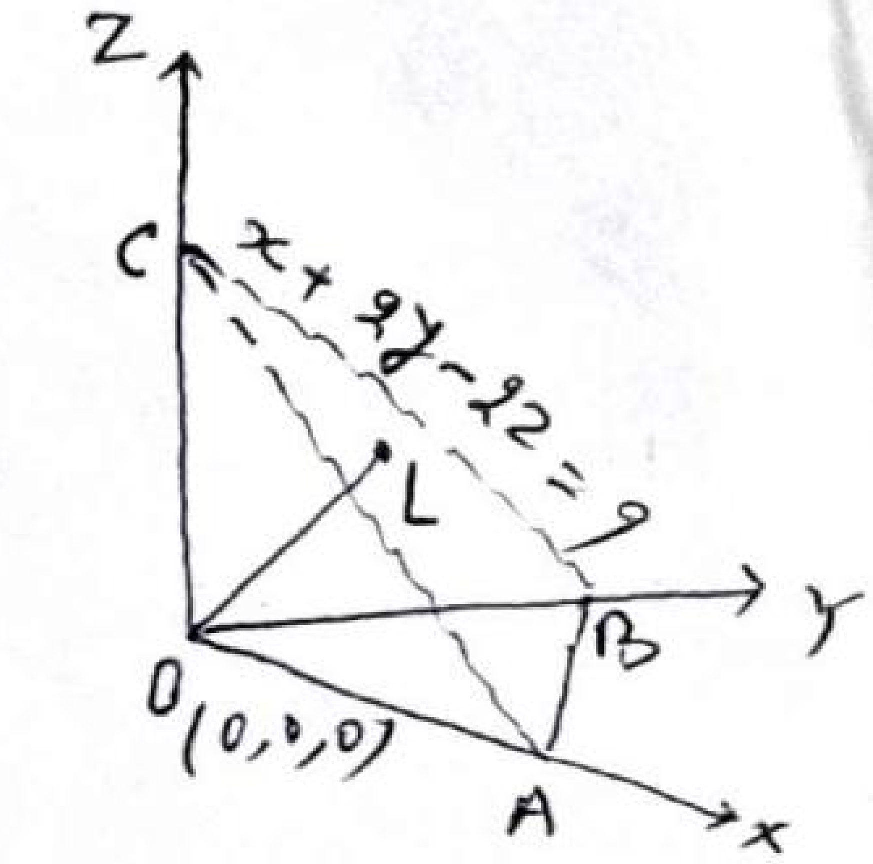
Now, θ is half the angle between the given planes. So that (1) bisects that angle between the planes which is $2\theta < 90^\circ$.

Hence the plane $2x + 7y - 9z = 6$ bisects the acute angle

Scanned by TapScanner

Problem-9 Find the intercepts made on the co-ordinate axes by the plane $x+2y-2z=9$

Find the length of the normal from the origin to the plane $x+2y-z=9$ and so the d.cs of the normal.



Sol^n:- Given $x+2y-2z=9$

or $\frac{x}{9}+\frac{y}{9/2}+\frac{z}{-9/2}=1$

which is of the form $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

$\left.\begin{array}{l} a=9 \\ b=9/2 \\ c=-9/2 \end{array}\right\}$ are the intercepts on the axes.

Now, From the given eq^n proportional d.cs of the normal to the given plane are $1, 2, -2$.

$\therefore$ Actual d.cs of the normals are $\frac{1}{\sqrt{1^2+2^2+(-2)^2}}, \frac{2}{\sqrt{1^2+2^2+(-2)^2}}, \frac{-2}{\sqrt{1^2+2^2+(-2)^2}}$

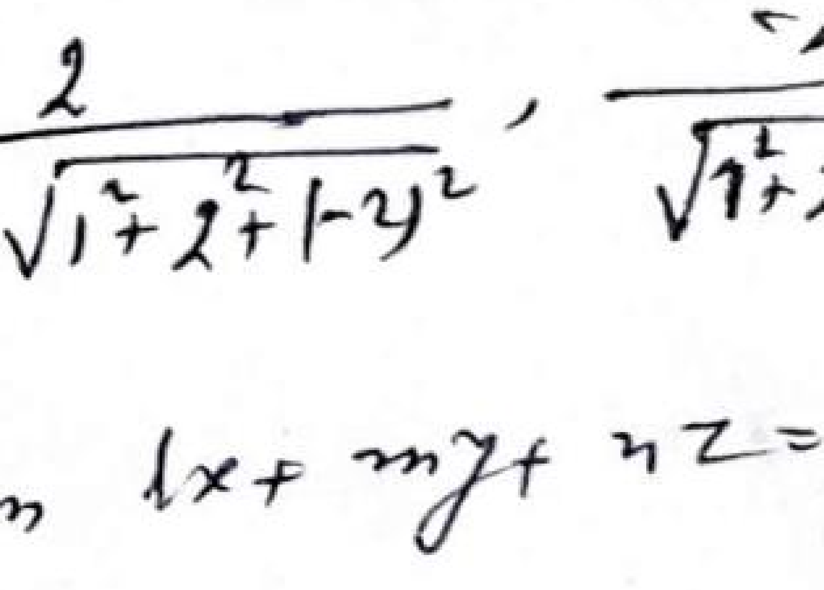
$=\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}$

Let us now write the normal form $lx+my+nz=p$. eq^n of the given plane in the normal form $\frac{x}{3}+\frac{2y}{3}-\frac{2z}{3}=\frac{9}{3} \Rightarrow \frac{x}{3}+\frac{2y}{3}-\frac{2z}{3}=3$.

So $\frac{x}{3}+\frac{2y}{3}-\frac{2z}{3}$

Hence the length of the normal from the origin to the plane is 3.

Problem-10 Find the eq^n to the plane through $P(2,3,-1)$ at right angles to OP, O being the origin.



Sol^n: The d.cs of OP are proportional to $(2-0, 3-0, -1-0)$ i.e. $(2,3,-1)$.

As OP is normal to the required plane, the equation to any plane whose normal is OP can be taken as $2x+3y-z+d=0$.

But the plane passes through $P(2,3,-1)$.

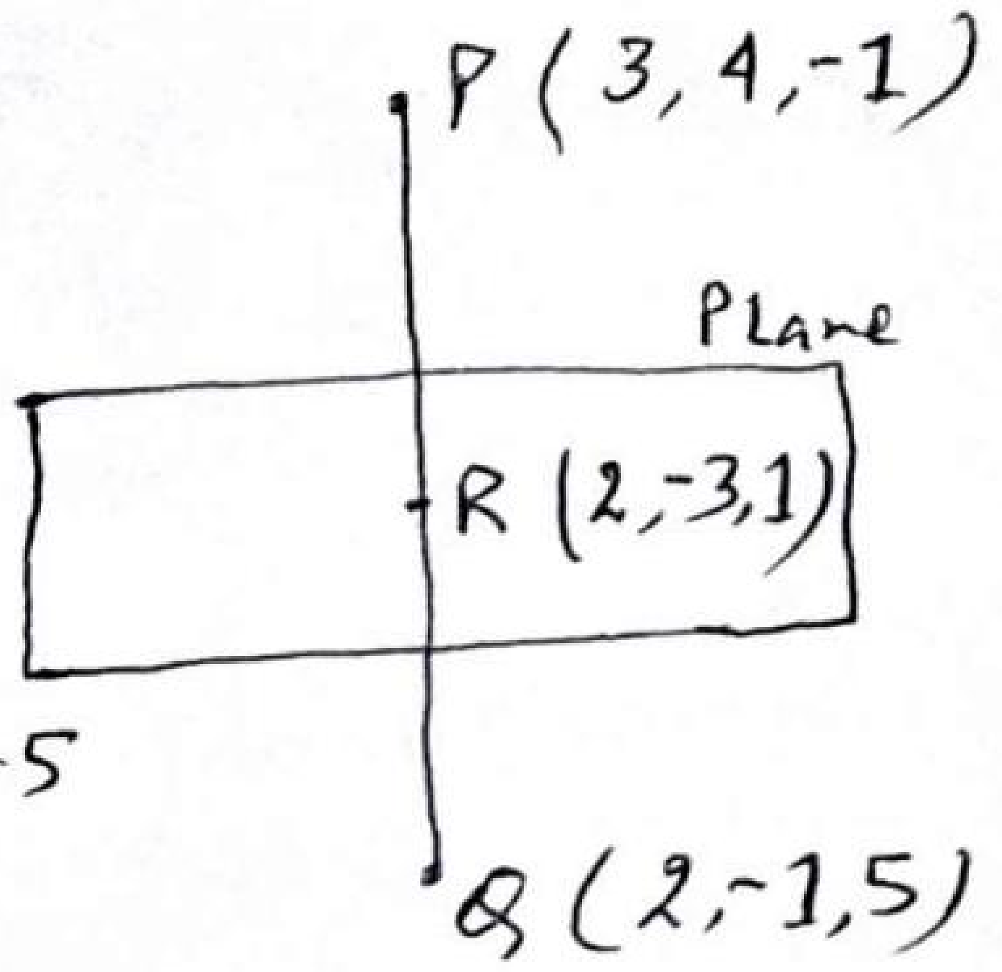
So $2(2)+3(3)-(-1)+d=0 \Rightarrow d=-14$

$\therefore$ the equation of the required plane is

$2x+3y-z-14=0$.

Scanned by TapScanner

Problem-11 Find the eqn of the plane that passes through $(2,-3,1)$ and is perpendicular to the line joining the points $(3,4,-1)$ and $(2,-1,5)$.



Soln: The plane is perpendicular to the line joining $P(3,4,-1)$ and $Q(2,-1,5)$. Proportional d.cs of the normal PQ to the given plane are $3-2,\ 4-(-1),\ -1-5$

i.e. $1, 5, -6$

Let the plane be $x+5y-6z+d=0$ from the formula $ax+by+cz+d=0$.

It passes through $(2,-3,1)$ so $d=19$

$\therefore$ Required plane is $x+5y-6z+19=0$

Problem-12 Show that the four points $(0,-1,-1)$, $(4,5,1)$, $(3,9,4)$ and $(-4,4,4)$ lie in a plane.

Soln:- Let the equation of the plane be $Ax+By+Cz+D=0$ — (1)

If it goes through the points $(0,-1,-1)$, $(4,5,1)$, $(3,9,4)$

so

$$-B-C+D=0 \quad \text{— (2)}$$
$$4A+5B+C+D=0 \quad \text{— (3)}$$
$$3A+9B+4C+D=0 \quad \text{— (4)}$$

(3) − (2) and (3) − (4)

$$\Rightarrow 4A+6B+2C=0$$
$$\Rightarrow 2A+3B+C=0$$
and $$A-4B-3C=0$$

Solving by cross-multiplication $\dfrac{A}{-9+4}=\dfrac{B}{1+6}=\dfrac{C}{-8-3}$

$$\Rightarrow \frac{A}{-5}=\frac{B}{7}=\frac{C}{-11}=K \text{ (say)}$$

$\therefore A=-5K,\ B=7K,\ C=-11K$

From (2) $D=-4K$

Hence, from (1), the equation of the plane is

$$5x-7y+11z+4=0$$

The fourth point $(-4,4,4)$ clearly lies on this plane.

Hence the given points are coplanar.

Scanned by TapScanner

Prob. 13 Prove that the points $(0,-1,0)$, $(2,1,-1)$, $(1,1,1)$, $(3,3,0)$ are coplanar

Prob. 14 Prove that the four points $(1,3,-1)$, $(3,5,1)$, $(0,2,-2)$, $(2,1,-2)$ are coplanar and find the eqn of the plane.

$\boxed{\text{Ans: } 20x + 23y + 26z - 69 = 0}$

Prob. 15 Find the eqn to the plane through the points $P(1,1,1)$, $Q(3,-1,2)$ and $R(-3,5,-4)$.

Soln:- Let the eqn of the plane be $ax + by + cz + d = 0$ — (1)

It passes through $P(1,1,1)$, $Q(3,-1,2)$ and $R(-3,5,-4)$

$\therefore\ a + b + c + d = 0$ — (2)

$3a - b + 2c + d = 0$ — (3)

$-3a + 5b - 4c + d = 0$ — (4)

(3) − (2) $\Rightarrow 2a - 2b + c = 0$ — (5)

(3) − (4) $\Rightarrow a - b + c = 0$ — (6)

Solving (5) & (6) $\dfrac{a}{-1} = \dfrac{b}{-1} = \dfrac{c}{0} = k$ (say)

$\Rightarrow a = -k,\ b = -k,\ c = 0$ | From (2) $d = 2k$

Put the value of a, b, c, d in (1)

$-kx - ky + 0 + 2k = 0 \Rightarrow x + y - 2 = 0$ is the required eqn of the plane.

Prob. 16 Find the eqn of the plane passing through the intersection of the planes $x + y + z = 6$ and $2x + 3y + 4z + 5 = 0$ and the pt $(1,1,1)$.

Soln: Any plane through the intersection of the given plane be

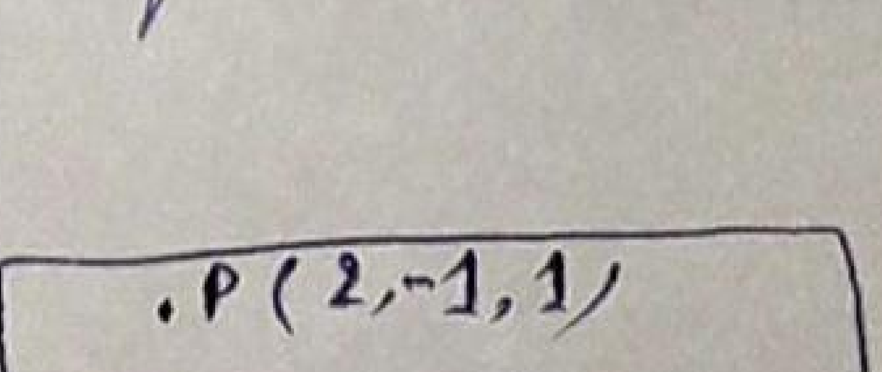
$x + y + z - 6 + k(2x + 3y + 4z + 5) = 0$

If it passes through $(1,1,1)$ so $k = 3/14$

Hence the required eqn of the plane is

$x + y + z - 6 + \frac{3}{14}(2x + 3y + 4z + 5) = 0 \Rightarrow 20x + 23y + 26z - 69 = 0$

Problem 17 Find the eqn of the plane through the point $(2,-1,+1)$ and the line $4x - 3y + 5 = 0 = y - 2z - 5$.



Soln:- Eqn to the plane through AB

$4x - 3y + 5 + k(y - 2z - 5) = 0$ — (1)

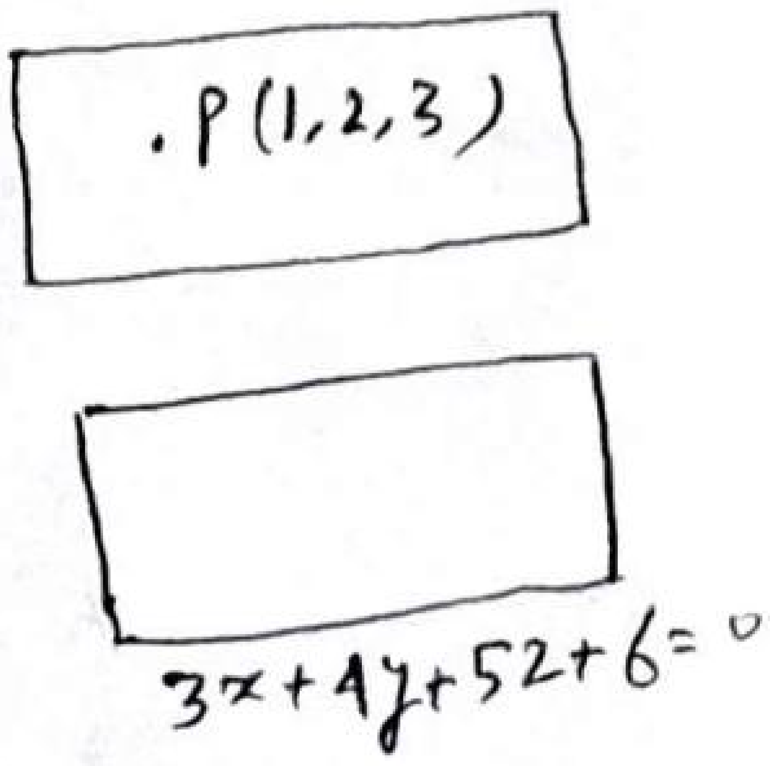
(1) passes through $P(2,-1,1)$ so $k = 2$

Put the value of k in (1) $4x - y - 4z - 5 = 0$,

which is the required eqn of the plane.

Scanned by TapScanner

Prob-18 Find the eqn to the plane through (1,2,3) parallel to $3x+4y+5z+6=0$



Soln: Eqn of the plane (given)

$$3x+4y+5z+6=0 \quad \text{—} \ (1)$$

Eqn of the plane parallel to (1) is

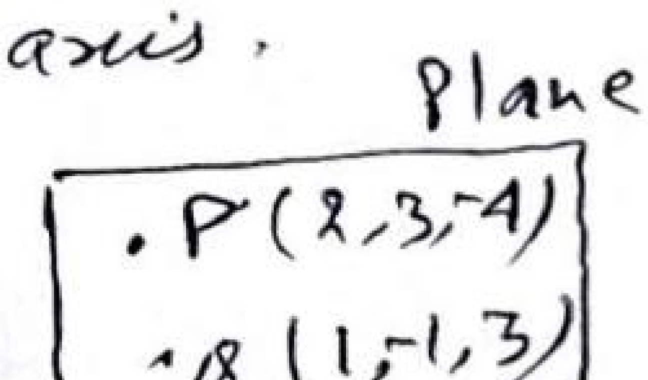
$$3x+4y+5z+K=0 \quad \text{—} \ (2)$$

Since (2) passes through (1,2,3), so

$$3+8+15+K=0 \Rightarrow K=-26$$

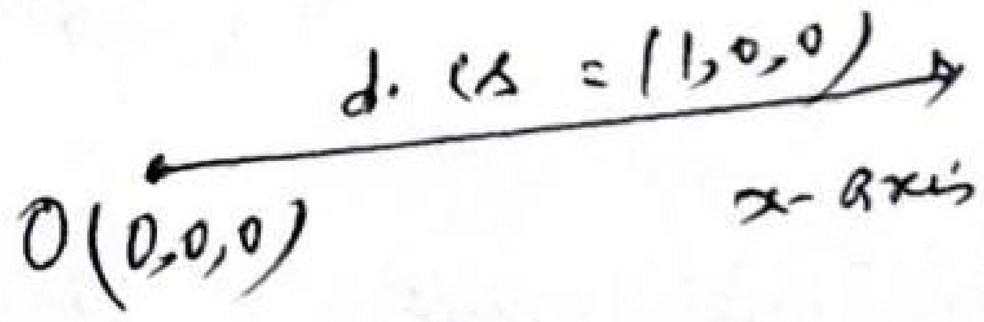
Put the value of K in (2) $3x+4y+5z=26$ is the required eqn of the plane.

Prob-19 Find the eqn of the plane through (2,3,−4) and (1,−1,3) and parallel to the x-axis.



Soln: Let the eqn of the plane be

$$ax+by+cz+d=0 \quad \text{—} \ (1)$$



It passes through (2,3,−4) and (1,−1,3)

$$2a+3b-4c+d=0 \quad \text{—} \ (2)$$

and $$a-b+3c+d=0 \quad \text{—} \ (3)$$

$$(2)-(3) \Rightarrow a+4b-7c=0 \quad \text{—} \ (4)$$

Since (1) is parallel to the x-axis whose d.cs are (1,0,0).

$$\therefore a.1+b.0+c.0=0 \Rightarrow a=0$$

Putting this value in (4) $4b-7c=0 \Rightarrow b=\frac{7}{4}c$

From (2), $2(0)+3\left(\frac{7}{4}c\right)-4c+d=0 \Rightarrow d=-\frac{5}{4}c$

From (1), $\frac{7}{4}cy+cz-\frac{5}{4}c=0 \Rightarrow 7y+4z-5=0$ is the required eqn of the plane

Prob. (20) Find the eqn of the plane through the point $(2,2,1)$ (6)
and $(9,3,6)$ and perpendicular to the plane $2x+6y+6z=9$

Soln: Eqn of the given plane is $2x+6y+6z=9$ — (1)
Let the eqn of the plane perpendicular to (1) be
$$ax+by+cz+d=0 \quad \text{— (2)}$$

$$\underset{\text{Formula}}{\boxed{aa_1+bb_1+cc_1=0}} \Rightarrow 2a+6b+6c=0 \Rightarrow a+3b+3c=0 \quad \text{— (3)}$$

From question, (2) passes through $(2,2,1)$ and $(9,3,6)$
$\therefore\ 2a+2b+c+d=0$ — (4) and $9a+3b+6c+d=0$ — (5)

(5) − (4) $\Rightarrow\ 7a+b+5c=0$ — (6)

Solving (3) & (6): $\dfrac{a}{30-6}=\dfrac{b}{42-10}=\dfrac{c}{2-42}$

$$\Rightarrow \frac{a}{3}=\frac{b}{4}=\frac{c}{-5}=k\text{ (say)}$$

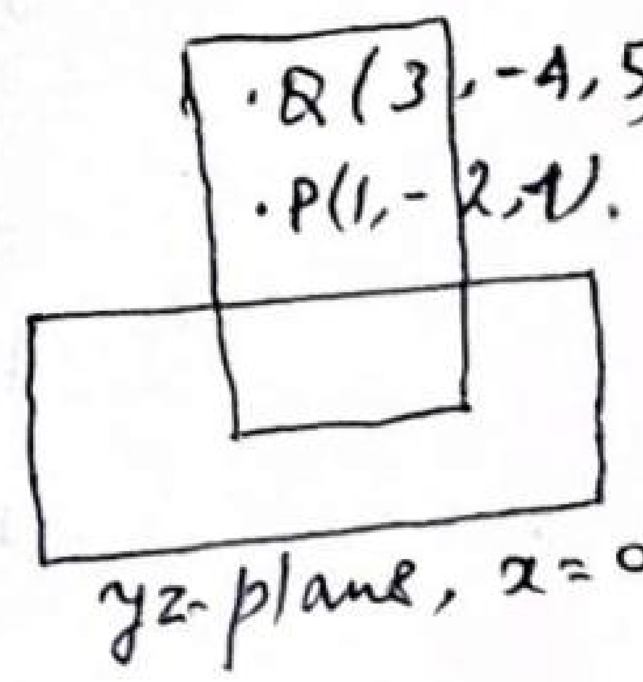
$\therefore\ a=3k,\ b=4k,\ c=-5k$

Put the value in (4) $\quad d=-9k.$

From (2) $\quad 3x+4y-5z=9$ is the required eqn of the plane.

Prob (21) Find the eqn of the plane through the points $(1,-2,4)$
and $(3,-4,5)$ and perpendicular to the yz-plane.

Soln: Eqn of the given, yz-plane, $x=0$ — (1)
Eqn of the plane perpendicular to (1) be
$$ax+by+cz+d=0 \quad \text{— (2)}$$



Then $\quad a.1+b.0+c.0=0 \Rightarrow a=0$ — (3)

Formula
$$\boxed{aa_1+bb_1+cc_1=0}$$

A/Q (2) passes through the points $(1,-2,4)$ and $(3,-4,5)$

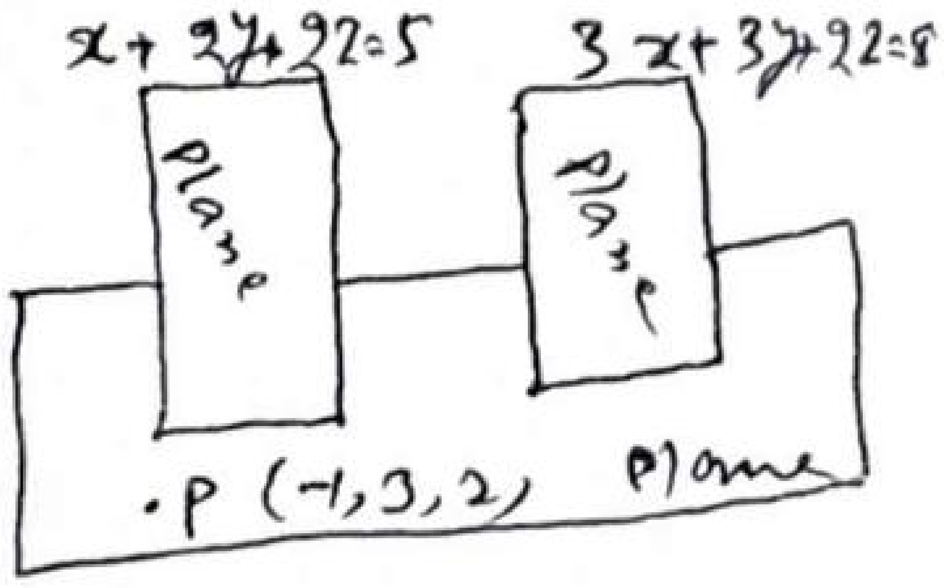
$\therefore\ a-2b+4c+d=0 \Rightarrow -2b+4c+d=0$ — (4)
and $3a-4b+5c+d=0 \Rightarrow -4b+5c+d=0$ — (5) $\Big\}$ using (3).

Solving (4) and (5) $\quad \dfrac{b}{4-5}=\dfrac{c}{-4+2}=\dfrac{d}{-10+16}$

$$\Rightarrow \frac{b}{-1}=\frac{c}{-2}=\frac{d}{6}=k\text{ (say)} \Rightarrow \left.\begin{array}{l} b=-k \\ c=-2k \\ d=6k \end{array}\right\}$$

$\therefore\ y+2z-6=0$ is the required eqn of the plane.

Prob. 22 Find the eq^n of the plane through the point $(-1,3,2)$ and perpendicular to the plane $x+2y+2z=5$ and $3x+3y+2z=8$.



Soln: Let the eq^n of the plane be

$$ax+by+cz+d=0 \quad \text{--- (1)}$$

If it passes through $(-1,3,2)$ then

$$-a+3b+2c+d=0 \quad \text{--- (2)}$$

Subtracting (2) from (1) $\quad a(x+1)+b(y-3)+c(z-2)=0 \longrightarrow (3)$

Since plane (3) is perpendicular to the given planes

$\therefore a+2b+2c=0$ and $3a+3b+2c=0$ } Solving these $\dfrac{a}{2}=\dfrac{b}{-4}=\dfrac{c}{3}=k$ (say).

$\therefore a=2k,\ b=-4k,\ c=3k.$

From (3) $\quad 2k(x+1)-4k(y-3)+3k(z-2)=0$

$\Rightarrow 2x-4y+3z+8=0$ is the required eq^n of the plane.

H.W. **Prob. 23** Find the eq^n of the plane through the point $(-4,3,7)$ and perpendicular to the planes $x+2y+2z=7$ and $3x+3y+2z=9$.

Ans: $\boxed{2x-4y+3z-1=0}$

Prob. 24 Find the equation of the plane through the intersection of the planes $x+2y+3z=4$ and $2x+y-z+5=0$ and perpendicular to the plane $5x+3y+6z+8=0$.

Soln: Any plane through the intersection of the given planes be

$$x+2y+3z-4+k(2x+y-z+5)=0$$

$$\Rightarrow (1+2k)x+(2+k)y+(3-k)z-(4-5k)=0 \quad \text{--- (1)}$$

The plane (1) is perpendicular to the given plane

$$5x+3y+6z+8=0$$

So $5(1+2k)+3(2+k)+6(3-k)=0 \Rightarrow k=-\dfrac{29}{7}.$

Put the value of k in (1), we get the required eq^n of the plane

$-51x-15y+50z-173=0 \Rightarrow 51x+15y-50z+173=0$