PHYSICS B.Sc Part-III (H) Paper-V Gr-C

By :- Dr. Mirza Abbas Hussain

5H

SCHRODINGER WAVE EQUATION OF FREE PARTICLE

The schrödinge equation of the particle is given by

$$\nabla^2\psi + \frac{2m}{\hbar}(E-V)\psi = 0 \quad \text{--- (1)}$$

For free particle $V=0$;

$$\therefore \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{2m}{\hbar}E\psi = 0 \quad \text{--- (2)}$$

Let solution of equ^n is given by

$$\psi(x,y,z) = X_x * Y_y * Z_z = XYZ \quad \text{--- (3)}$$

Where X_x, Y_y, Z_z are the fuctions along x, y and z-axes.

Now $\frac{\partial\psi}{\partial x} = \frac{\partial X}{\partial x}YZ$ & $\frac{\partial^2\psi}{\partial x^2} = \frac{\partial^2 X}{\partial x^2}YZ$

and $\frac{\partial^2\psi}{\partial y^2} = XZ\frac{\partial^2 Y}{\partial y^2}$ and $\frac{\partial^2\psi}{\partial z^2} = XY\frac{\partial^2 Z}{\partial z^2}$

From equ^n (2)

$$YZ\frac{\partial^2 X}{\partial x^2} + ZX\frac{\partial^2 Y}{\partial y^2} + XY\frac{\partial^2 Z}{\partial z^2} + \frac{2m}{\hbar}EXYZ = 0$$

Deviding by XYZ

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} + \frac{2m}{\hbar}E = 0 \quad \text{--- (4)}$$

In this equ^n each term is a constant quantity

$$\therefore \frac{1}{X}\frac{\partial^2 X}{\partial x^2} = K_x, \quad \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = K_y$$

$$\& \quad \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = K_z.$$

Equ.^{n} (4) becomes

$$K_x + K_y + K_z = -\frac{2mE}{\hbar^2}$$

Let us put

$$K_x = -\frac{2mE_x}{\hbar^2},\ K_y = -\frac{2mE_y}{\hbar^2}\ \&\ K_z = -\frac{2mE_z}{\hbar^2}$$

$$\therefore \frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -\frac{2mE_x}{\hbar^2}$$

or, $$\frac{\partial^2 X}{\partial x^2} + \frac{2mE_x}{\hbar^2} X = 0 \quad (5)$$

The general solution of this equation is

$$X(x) = N_x \sin\left[\frac{1}{\hbar}\sqrt{2mE_x}\,(x - x_0)\right] \quad (6)$$

Similarly $Y(y) = N_y \sin\left[\frac{1}{\hbar}\sqrt{2mE_y}\,(y - y_0)\right]$

& $Z(z) = N_z \sin\left[\frac{1}{\hbar}\sqrt{2mE_z}\,(z - z_0)\right]$

Now,
$$\psi(x,y,z) = N \sin\left[\frac{1}{\hbar}\sqrt{2mE_x}\,(x-x_0)\right] \cdot \sin\left[\frac{1}{\hbar}\sqrt{2mE_y}\,(y-y_0)\right] \sin\left[\frac{1}{\hbar}\sqrt{2mE_z}\,(z-z_0)\right] \quad (7)$$

where $N = N_x N_y N_z$ = Normalisation constant. For the physical interpretation of wave function let $E_y = E_z = 0$ & $E_x = E$

$$\therefore \psi(x,y,z) = N \sin\left[\frac{1}{\hbar}\sqrt{2mE}\,(x-x_0)\right] \quad (8)$$

For the time dependent wave function

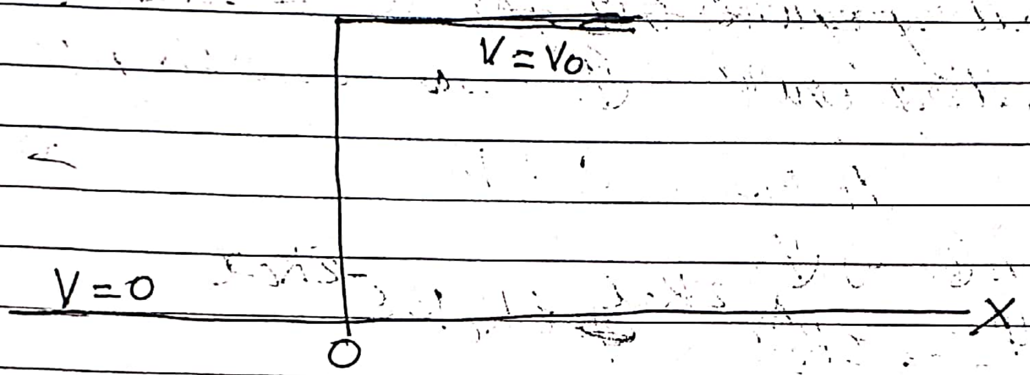
$$\psi(r,t) = N \sin \frac{1}{\hbar}\sqrt{2mE}\,(x-x_0)\, e^{iEt/\hbar} \quad (9)$$

This represents a set of standing waves with wavefront normal to the x-axis. The wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mE}}$$

Where $E = \frac{1}{2}mv^2$ = Energy Eigen value.

SCHRÖDINGER'S WAVE EQUATION for POTENTIAL STEP



For Potential Step, $V_x = 0$ for $x < 0$
$V_x = V_0$ for $x > 0$

The time independent S.E is given by

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{---} \quad (1)$$

For $x < 0$, $$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \quad \text{---} \quad (2)$$

For $x > 0$. $$\frac{d^2\psi}{dx^2} + \frac{2m(E-V_0)}{\hbar^2}\psi = 0 \quad \text{---} \quad (3)$$

or, For $x < 0$ $$\frac{d^2\psi}{dx^2} + K_0^2\psi = 0 \quad \text{---} \quad (4)$$

and for $x > 0$ $$\frac{d^2\psi}{dx^2} + K^2\psi = 0 \quad \text{---} \quad (5)$$

Where $K_0 = \sqrt{2mE}/\hbar$ & $K = \frac{\sqrt{2m(E-V_0)}}{\hbar}$